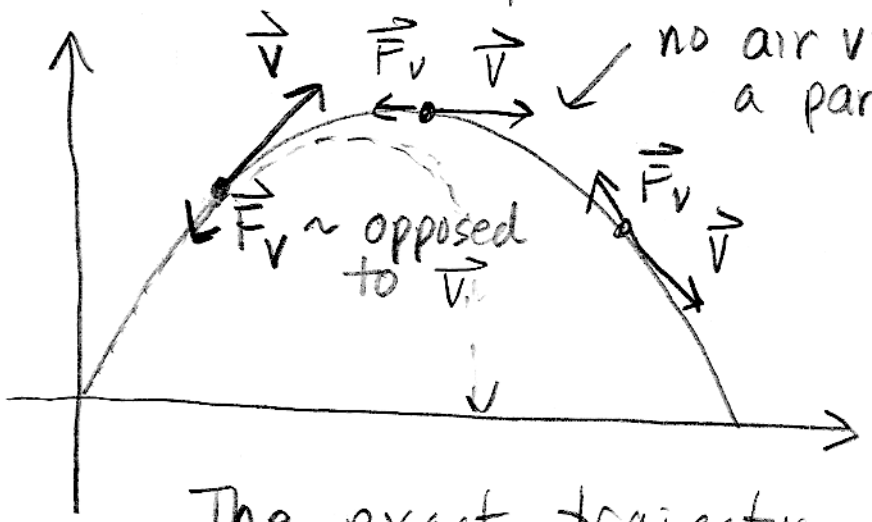
Introductory Differential Equations

Viscosity : $\vec{F}_v = -C\vec{v}$

"drag force" → examples are air resistance
water resistance

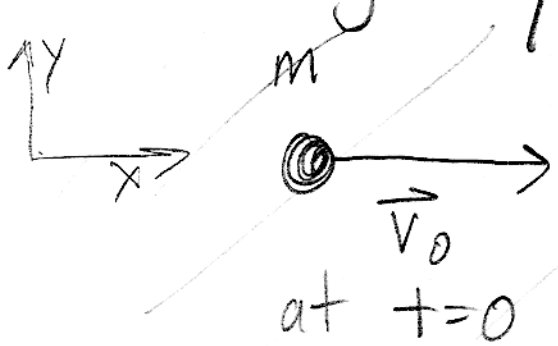
Note: **opposes** velocity, whichever direction velocity points.

Example: trajectory on earth



The exact trajectory is hard to compute, because the **direction** of $\vec{F}_v$ changes.

Imagine motion in outer space, with no gravity



← material, coefficient of viscosity C (must be given)

$$m a_x = \boxed{m \frac{dv_x}{dt} = -C v_x}$$

- a differential equation
- first order → means, generally, <u>one</u> arbitrary constant involved
  second order – <u>two</u> arbitrary constants
  etc.
- <u>initial conditions</u> or <u>final conditions</u> used to get the arbitrary constants

<u>note</u>:

$$\frac{dv_x}{v_x} = -\frac{C}{m} dt$$

$$\int_{v_x(0)}^{v_x(t)} \frac{dv_x}{v_x} = \ln v_x \Big|_{v_0}^{v_x(t)} = \int_0^t -\frac{C}{m} dt = -\frac{C}{m} t$$

initial condition → $v(0) = v_0$

$$\ln\left(\frac{v_x(t)}{v_0}\right) = -\frac{C}{m} t$$

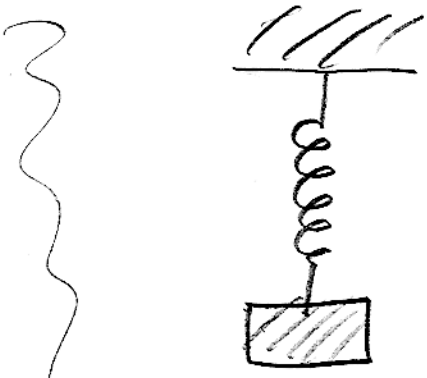
$$\frac{v_x(t)}{v_0} = e^{-\frac{C}{m} t}$$

$$\boxed{v_x(t) = v_0 e^{-\frac{C}{m} t}}$$

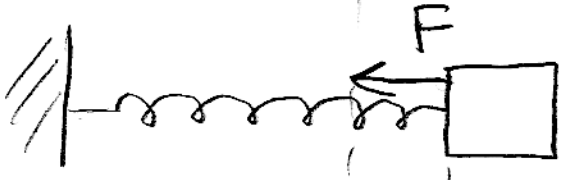
"exponential decay"

The Spring

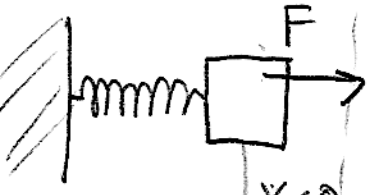
equilibrium position (no force)

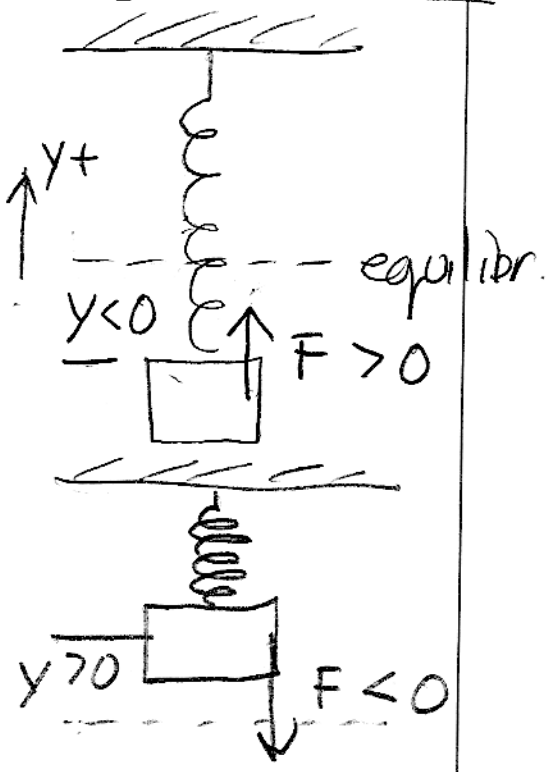


equilibrium position → some force, but balances gravity.



$x > 0$, $F < 0$
restoring force





$$F = -kx$$

Restoring; displacement

depends on spring: k bigger, spring stiffer

$k > 0$

so, $\boxed{m\frac{d^2x}{dt^2} = -kx}$

- simple harmonic oscillator.
- x = displacement from equilibrium
- expect two arbitrary constants

try: $x = x_0 e^{\alpha t}$

$\frac{dx}{dt} = \alpha x_0 e^{\alpha t}$

$\frac{d^2x}{dt^2} = \alpha^2 x_0 e^{\alpha t}$

$m\alpha^2 \cancel{x_0 e^{\alpha t}} = -k\cancel{x_0 e^{\alpha t}}$

$\alpha^2 = -\frac{k}{m}$ uh-oh...

$\alpha = \pm\sqrt{-\frac{k}{m}} = \pm i\cdot\frac{k}{m}$ no go... (will work later)

[great equation $e^{i\pi} + 1 = 0$]

recall:

$\frac{d}{dt}(\sin t) = \cos t$

$\frac{d^2}{dt^2}(\sin t) = -\sin t$

$\frac{d^2}{dt^2}(\cos t) = -\cos t$

try $x(t) = A\sin\alpha t + B\cos\alpha t$

two constants!