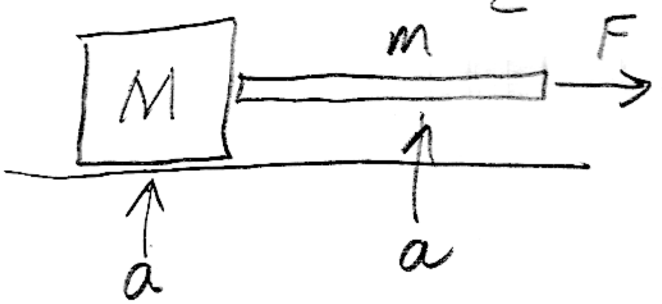
Massive String



known: M, m, F
unknown: a, F'

frictionless

① the string

$$\overset{F'}{\longleftarrow}\ \overset{F}{\longrightarrow} \qquad \uparrow y \ \ \rightarrow x$$

$$F - F' = ma$$

② the mass M

$$\longrightarrow F' \quad (\text{Newton III})$$

$$F' = Ma$$

$$F = (m+M)a$$

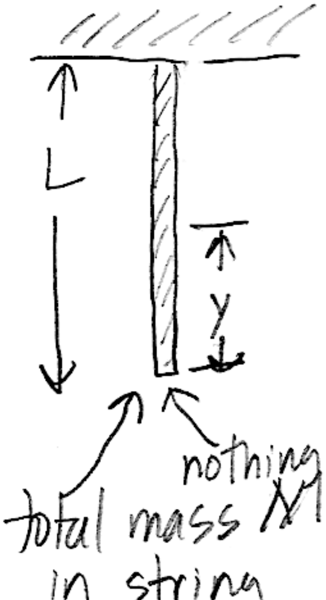
$$\boxed{a = \frac{F}{(m+M)}} \qquad m \ll M \quad a \cong F/M$$

$$F - F' = \frac{F}{(m+M)} \times m$$

$$F\left(1 - \frac{m}{m+M}\right) = F'$$

$$\boxed{F' = \frac{M}{m+M} F} \qquad m \ll M \quad F' \simeq F$$

If string is uniform in mass density –



assuming mass density uniform

mass in an interval y

$$= \left(\frac{y}{L}\right) M$$

Draw forces on that mass in the interval:

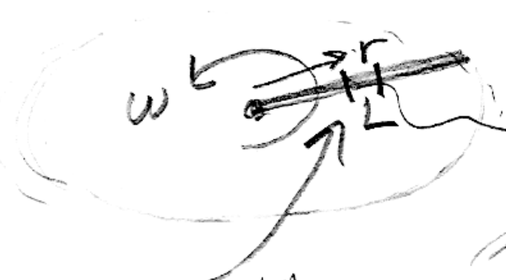
$\uparrow T(y)$

$\downarrow \left(\frac{y}{L}\right) Mg$

$$T(y) - \left(\frac{y}{L}\right) Mg = 0$$

$$\boxed{T(y) = \left(\frac{y}{L}\right) Mg}$$

Whirling Rope (big fun)

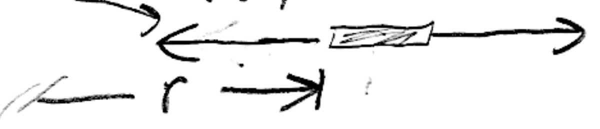


mass M

ignore gravity

mass density: $\frac{M}{L}$

Small segment of rope:



$$T(r+\Delta r) - T(r) = \left(\frac{M}{L}\Delta r\right) \cdot (-a$$

centripetal acceleration $= \omega^2 r$

$$\Delta T = \frac{-M r \omega^2}{L} \cdot \Delta r$$

calculus:

$$\frac{dT}{dr} = \frac{-M r \omega^2}{L}$$

$$T(r) = \frac{-1}{2}\frac{M\omega^2}{L} r^2 + \text{constant}$$

<u>constant</u>: when $r = L$, $T(L) = 0$

nothing pulling on the last bit of rope

so, $T(L) = 0 = \frac{-1}{2}\frac{M\omega^2}{L} \cdot L^2 + \text{constant}$

$$\text{constant} = \frac{1}{2}\frac{M\omega^2}{L} L^2$$

$$\boxed{T(r) = \frac{1}{2}\frac{M\omega^2}{L}\left(L^2 - r^2\right)}$$

<u>Friction</u>

- depends on Normal force
- $\perp$ to Normal force
- Resists applied force
- when body <u>at</u> <u>rest</u>,

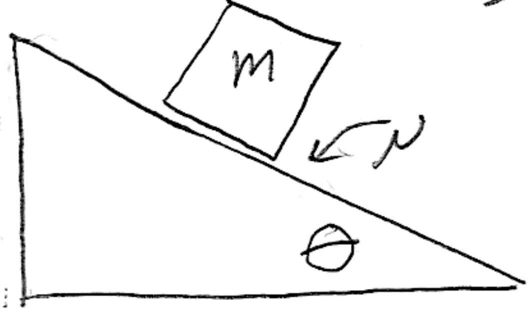
f=friction force

$$0 \le f \le \mu N$$

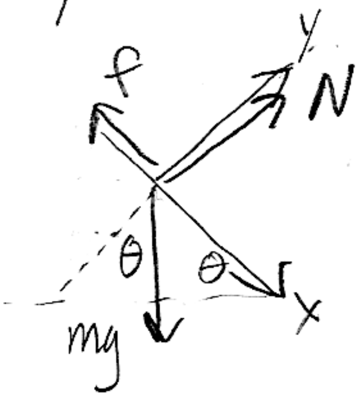
and f is always unknown

- when body no longer at rest

$f = \mu N$, <u>opposes</u> motion



as θ increases, when does the mass slip?



y: $N - mg\cos\theta = 0$

$N = mg\cos\theta$

note: as $\theta\uparrow$ $N\downarrow$

x: $-f + mg\sin\theta = 0 \quad (ma_x)$

sticking   sliding

which?

assume $f = \mu N$

$-\mu N + mg\sin\theta \geq 0$

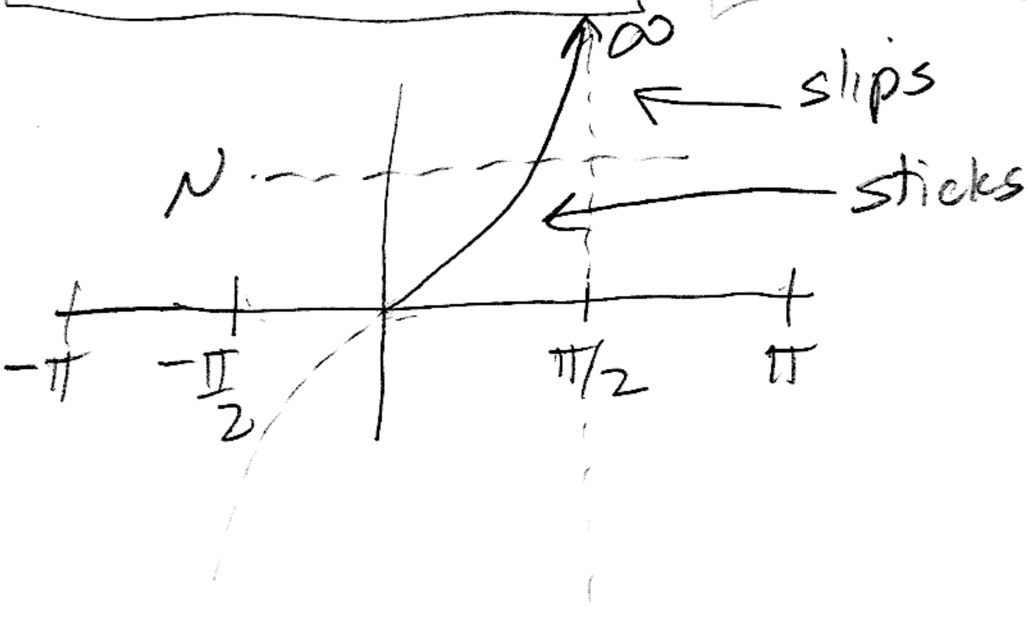
$-\mu mg\cos\theta + mg\sin\theta \geq 0$

$\sin\theta > \mu\cos\theta$

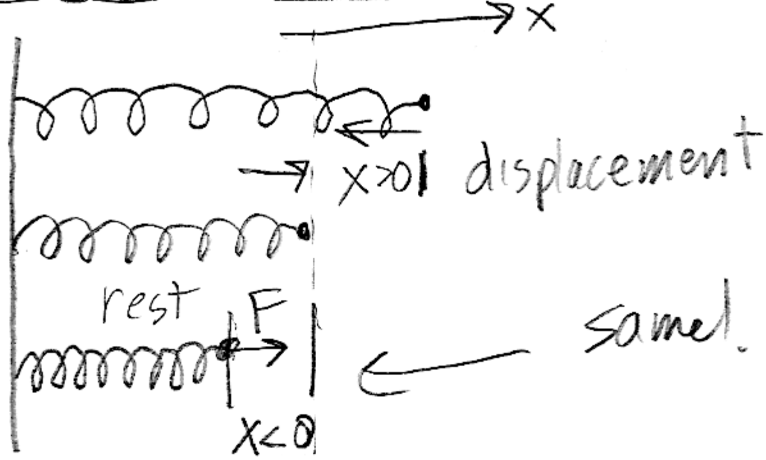
$\boxed{\tan\theta \geq \mu \text{ for slipping}}$

remember: friction only "brakes" never pushes

$\boxed{\tan\theta < \mu \text{ sticking}}$



Springs – first look



$F \propto -x$
$= -kx$

same!

spring

now $\quad -kx = ma_x = m\ddot{x} = m\dfrac{d^2x}{dt^2}$

or $\quad \ddot{x} + \left(\dfrac{k}{m}\right)x = 0$

try $x = A\sin(\omega t) + B\cos(\omega t)$

$\dot{x} = +\omega A\cos(\omega t) - \omega B\sin(\omega t)$

$\ddot{x} = -\omega^2 A\sin(\omega t) - \omega^2 B\cos(\omega t)$

$= -\omega^2\left(A\sin(\omega t) + B\cos(\omega t)\right)$

$\ddot{x} = -\omega^2 x$

$\ddot{x} + \omega^2 x = 0$

works as long as $\underline{\omega^2 = \dfrac{k}{m}}$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$