Trajectories

Forces $\longleftrightarrow$ **acceleration**:
- integrate to get velocity.
- integrate again to get position.

↑
gravity

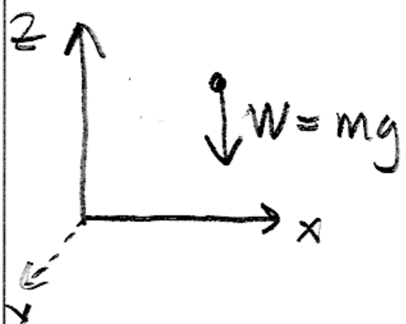
Newton II:

$$\sum_i \vec{F}_i = \vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{x}}{dt^2}$$
$$= m\ddot{\vec{x}} = m\dot{\vec{v}}$$

really <u>3</u> equations

$$F_{net,x} = ma_x = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2}$$
$$F_{net,y} = ma_y = m\frac{dv_y}{dt} = m\frac{d^2y}{dt^2}$$
$$F_{net,z} = ma_z = m\frac{dv_z}{dt} = m\frac{d^2z}{dt^2}$$

In a uniform gravitational field... falling objects:



Ignoring air resistance,

$$0 = ma_x = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2}$$

$$\boxed{v_x = \text{constant} = v_{0x}}$$

↑ initial

$$v_x = \frac{dx}{dt} = v_{0x} \rightarrow x = v_{0x}t + \text{constant}$$

$$\boxed{x = v_{0x}t + x_0}$$

What happens in the x coordinate is independent of what happens in y, or z. VERY IMPORTANT

$$\boxed{y = v_{0y}t + y_0} \qquad \boxed{v_y = v_{0y}}$$

z: special. Constant, downward, wherever particle is.

$$-mg = ma_z = m\frac{dv_z}{dt}$$

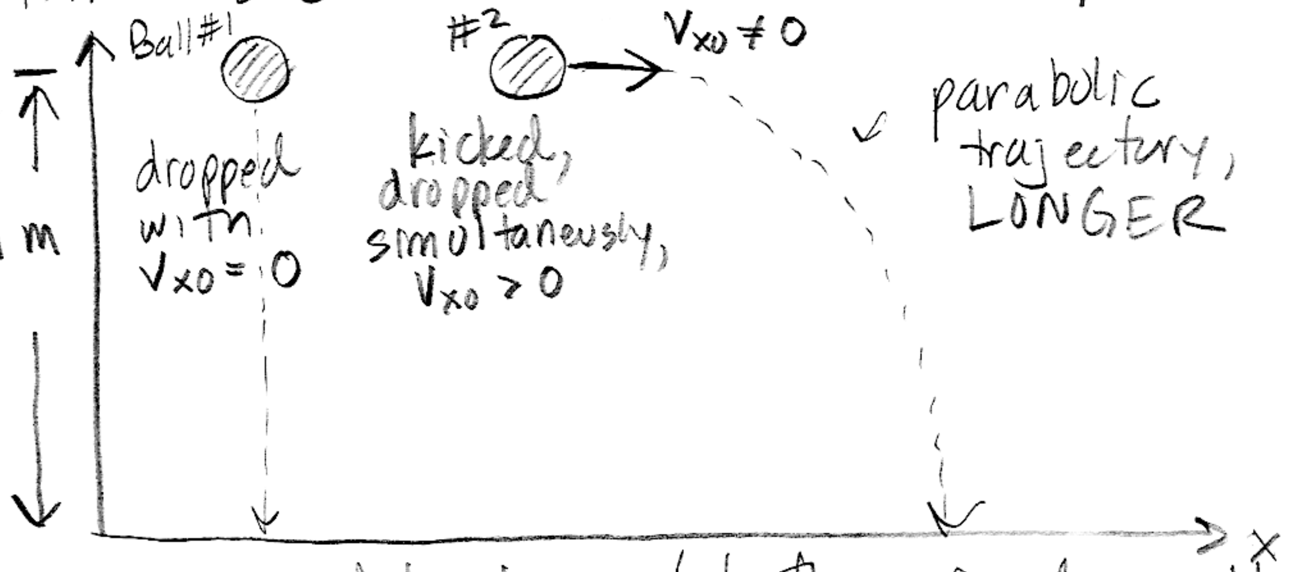
integrate $\frac{dv_z}{dt} = -g$

$$v_z = -gt + \text{constant} \Rightarrow \begin{matrix} t=0 \\ v_z = v_{z0} \end{matrix}$$

so $v_z = -gt + v_{z0}$

$$\boxed{z = -\frac{1}{2}gt^2 + v_{z0}t + z_0}$$

These equations describe, parametrically, PARABOLIC TRAJECTORIES. Compare...



but, they hit the ground simultaneously

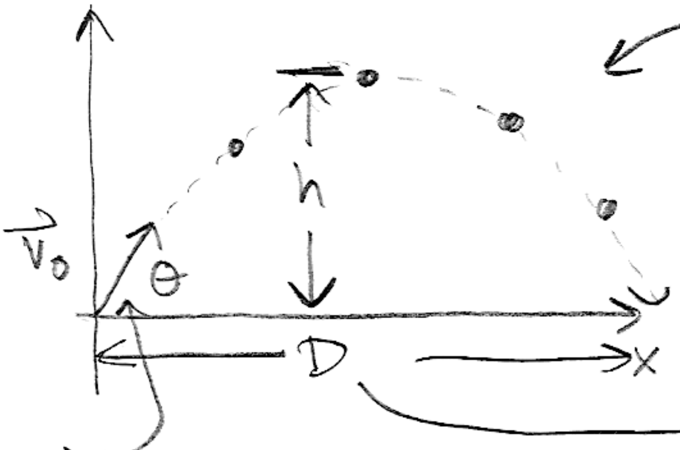
why, both cases share the same z-equation .... $z_0 = 1m$, $v_{z0} = 0$

$z = z_0 - \frac{1}{2}gt^2$ ... time when ball hits ground

$z_0 - \frac{1}{2}gt_G^2 = 0$

$t_G = \sqrt{\frac{2z_0}{g}} = \sqrt{\frac{2 \cdot 1}{9.8}}$

$= 0.45s$



does: acceleration ever change?

<u>NO</u>

- velocity does
- position does

want to <u>maximize D</u>

how to choose θ?

$\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$

$= v_0(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$

suppose this is fixed

suppose θ can be varied.

x: $x = v_{0x}t + \cancel{x_0}\,0$ by choice $= v_0\cos\theta \cdot t$

z: $z = -\frac{1}{2}gt^2 + v_{z0} \cdot t + \cancel{z_0}\,0$ by choice

$= -\frac{1}{2}gt^2 + v_0\sin\theta\, t = 0$

$t\left(v_0\sin\theta - \frac{1}{2}gt\right) = 0$

t=0 is initial

later

from z: $t = \frac{2v_0}{g}\sin\theta$ / plug into x

use x

$$D = v_0\cos\theta \cdot \left(\frac{2v_0}{g}\right)\sin\theta$$

$$\boxed{D = \frac{2v_0^2}{g}\sin\theta\cos\theta = \frac{v_0^2}{g}\cdot\sin 2\theta}$$

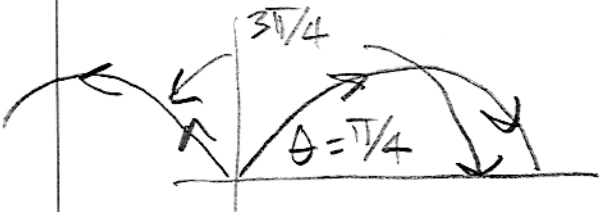
$\propto v_0^2$

1) maximum when $2\theta = \frac{\pi}{2}$

$\theta = \pi/4$

2) proof: $\frac{d}{d\theta}(\sin 2\theta) = 2\cos 2\theta = 0$

when $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \cancel{\frac{5\pi}{4}}$ negative



$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

3) at maximum, $D = \frac{v_0^2}{g}$

say, $v_0 = 100$ mph (good baseball pitcher)

$$= 100 \cdot 1.61 \frac{km}{mile} \cdot 10^3 \frac{m}{km} \cdot \frac{1}{60}\frac{h}{min} \cdot \frac{1}{60}\frac{min}{sec}$$

$v_0 = 44.7$ m/s

$$D = \frac{44.7^2\ m^2/s^2}{9.8\ m/s^2} = 204 \text{ meters}$$

however, <u>wind</u> <u>resistance</u> <u>limits</u>.