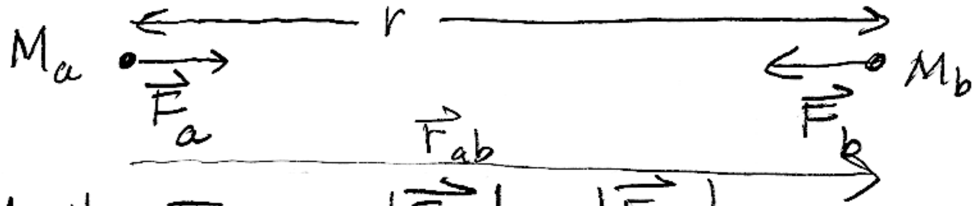
Gravitational Force

point masses M_a and M_b, separated by distance r



Newton III : $|\vec{F}_a| = |\vec{F}_b|$

Gravitation: ① Force is attractive, and along a line joining the masses.

$\vec{r}_{ab}$ = vector from a to b

$\vec{r}_{ba}$ = vector from b to a

$= -\vec{r}_{ab}$

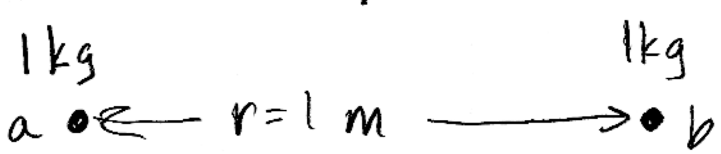
$\hat{r}_{ab}$ = unit vector in same direction as $\vec{r}_{ab}$

$\hat{r}_{ba} = -\hat{r}_{ab}$

② Magnitude of $|\vec{F}_a|$

$$|\vec{F}_a| = \frac{G M_a M_b}{r^2}$$

G = constant $= 6.67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$



$$|\vec{F}_a| = |\vec{F}_b| = 6.67 \cdot 10^{-11} \cdot \frac{1 \cdot 1}{1^2} \cancel{\frac{kg^2}{m^2}} \cdot \frac{N \cancel{m^2}}{\cancel{kg^2}}$$

$$= 6.67 \cdot 10^{-11}\ N$$

$m_a g = 1\,kg \cdot 9.8 \frac{m}{s^2} = 9.8\,N \simeq 10^{12} \times |F_a|$

Earth's mass makes its gravity overwhelm.

acceleration of a mass under the force of gravity is independent of its mass.

$$\vec{F}_b = -\frac{G M_a M_b}{r^2}\hat{r}_{ab} = M_b \cdot \vec{a}_b$$

$$\vec{a}_b = -\frac{G M_a}{r^2}\hat{r}_{ab}$$

independent of M_b

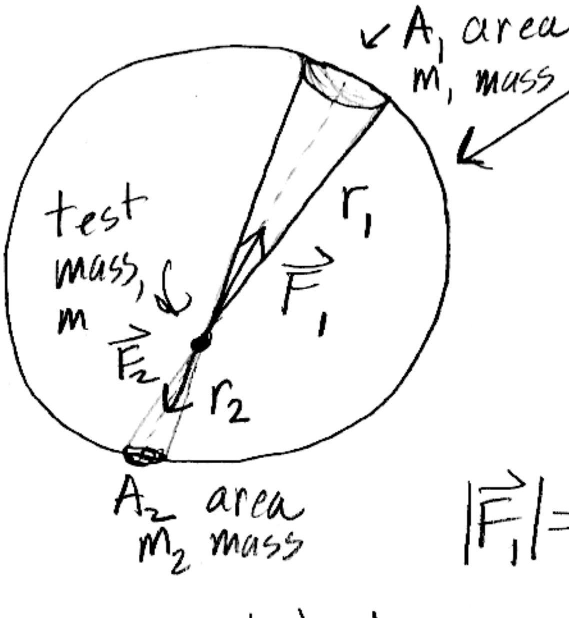
But... do they really, really cancel? topic of research

$$\left(-\frac{G M_a}{r^2}\hat{r}_{ab}\right)\cdot \underline{\underline{M_b^G}} \stackrel{?}{=} \vec{a}_b \times \underline{M_b^I}$$

"Gravitational Mass" "inertial mass"

does M_b^G really $= M_b^I$?

What about an extended object (not a point)?



uniform, spherical shell.

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{m_1}{m_2}$$

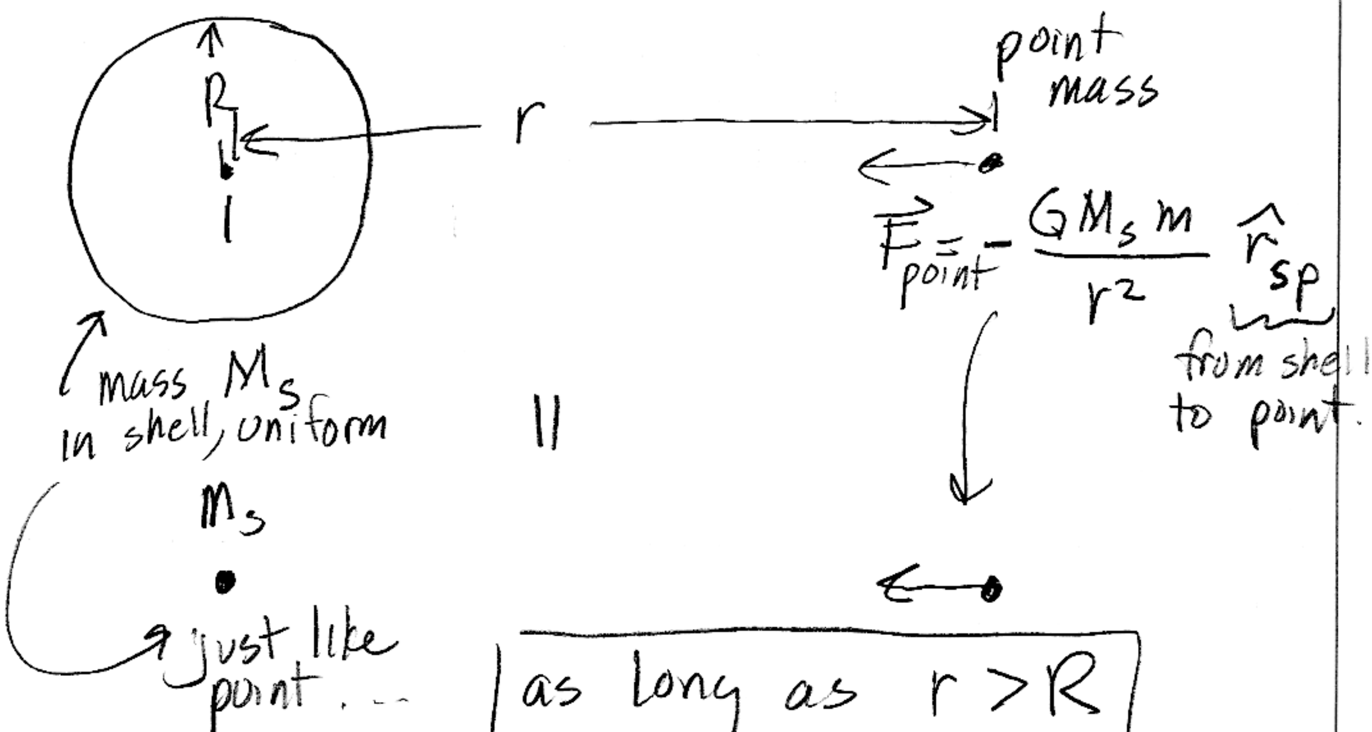
so $\dfrac{m_1}{r_1^2} = \dfrac{m_2}{r_2^2}$

$$|\vec{F}_1| = G\frac{m m_1}{r_1^2} = \frac{G m m_2}{r_2^2} = |\vec{F}_2|$$

but these are oppositely directed.

No net force anywhere within a uniform shell of mass, due to gravity

Outside, force same as if all was concentrated at center!

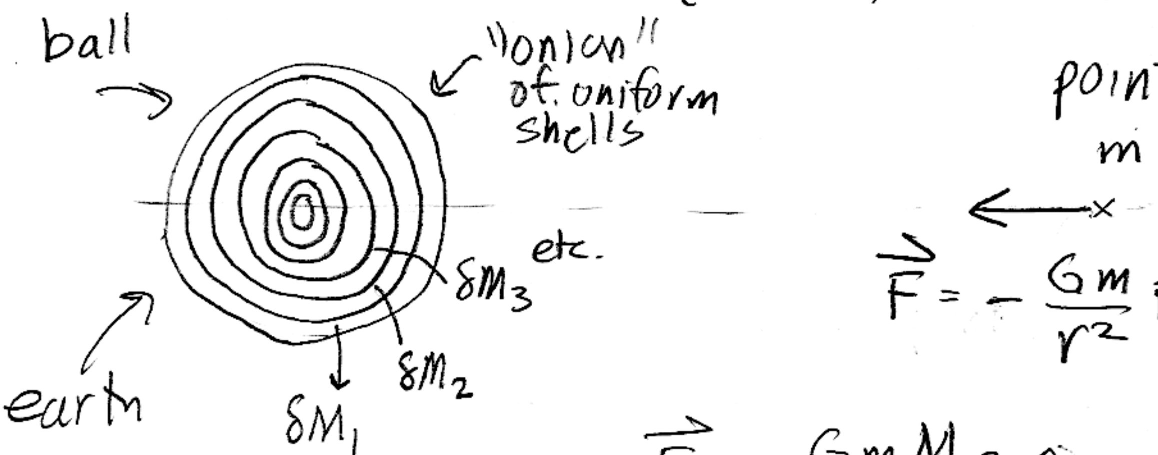


$$\vec{F}_{point} = -\frac{G M_s m}{r^2}\,\underbrace{\hat{r}_{sp}}$$

$$\boxed{\text{as long as } r > R}$$

$r < R$: $\vec{F}_{point} = 0$.

Outside the Earth (or sun, saturn)



$$\vec{F} = -\frac{G m}{r^2}\,\hat{r}_{bp} \cdot \underbrace{\sum \delta M_i}_{\text{total mass of earth}}$$

$$\vec{F} = -\frac{G m M_e}{r^2}\,\hat{r}_{bp}$$

as long as point p is <u>outside</u> the earth.

Earth's surface: $\cancel{m} g = \dfrac{G \cancel{m} M_e}{R_e^2}$

$$g = G\frac{M_e}{R_e^2} = 9.8\ m/s^2$$

$$R_e = 6.37\cdot 10^6\ m$$ (actually, known from measurements in Egypt in antiquity, Eratosthenes)

so, $$M_e = \frac{R_e^2}{G}\cdot g = \frac{(6.37\cdot 10^6)^2}{6.67\cdot 10^{-11}}\cdot 9.8$$ (meters, seconds, kg)

$$\boxed{M_e = 5.96\cdot 10^{24}\ kg \simeq 10\ \text{moles of kg}}$$

g on other planets, moons

	g	W = 130 lbs, earth
Moon	0.41 m/s^2	5.4 lbs
Io	0.45 m/s^2	6.0
Titan	0.34 m/s^2	4.5
Mercury	3.70 m/s^2	49.
Venus	8.87	117.7
Mars	3.71	49.2
Jupiter	23.12	306.7
Saturn	8.96	118.9
Uranus	7.77	103.1
Neptune	11.0	145.9
Pluto	0.07	0.93

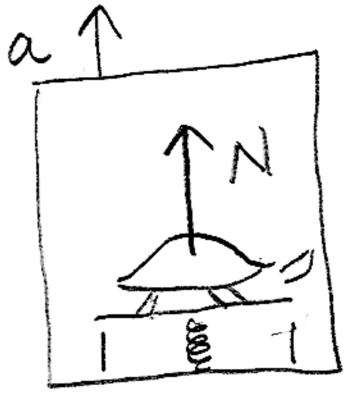
$$W = mg$$

$$\left.\begin{aligned} W_{earth} &= m\cdot g_{earth} \\ W_x &= m\cdot g_x \end{aligned}\right\rangle \quad \frac{W_x}{W_{earth}} = \frac{g_x}{g_{earth}}$$

$$W_x = \left(\frac{g_x}{g_{earth}}\right)\cdot W_{earth}$$

Scales measure normal force, not weight

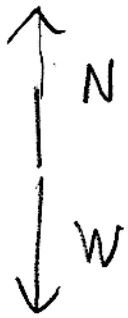
(W = N when no acceleration)



← Turtle in elevator accelerating upward with acceleration a.

N = normal force of scale on turtle.

(N III: = force of turtle on scale)



$$N - W = m_t \cdot a$$

$$N - m_t \cdot g = m_t \cdot a$$

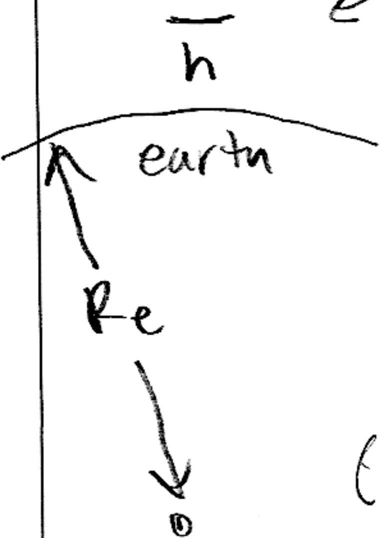
$$N = m_t (g + a)$$

on the scale: $m_t (g+a)$

Variation of Gravity with altitude

$$F_g = G \frac{M_e}{(R_e+h)^2} \cdot m$$

$$= G \frac{M_e}{R_e^2} \cdot \left(\frac{R_e}{R_e+h}\right)^2 \cdot m$$



$$\boxed{F_g = m g \times \left(\frac{1}{1 + h/R_e}\right)^2}$$

p.41

$$\frac{1}{\left(1+\frac{h}{R_e}\right)^2} \simeq 1 - 2\frac{h}{R_e} + \frac{(-2)(-3)}{2}\left(\frac{h}{R_e}\right)^2 + \ldots$$

$$= 1 - 2\frac{h}{R_e} + 3\left(\frac{h}{R_e}\right)^2 + \ldots$$