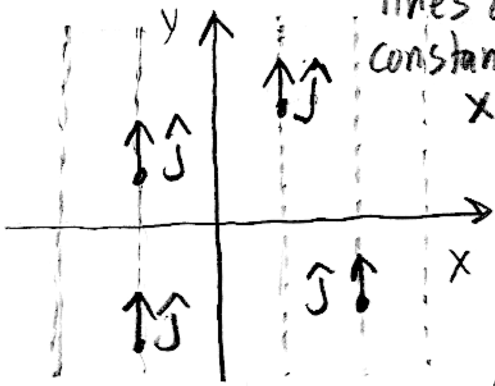
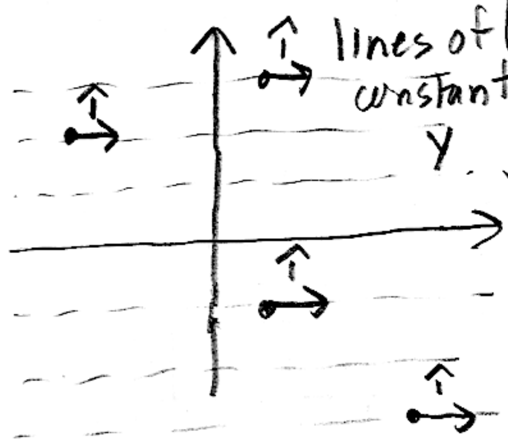
Cylindrical Coordinates

z stays same, but describe x-y plane

"Rectilinear" "Cartesian" x-y

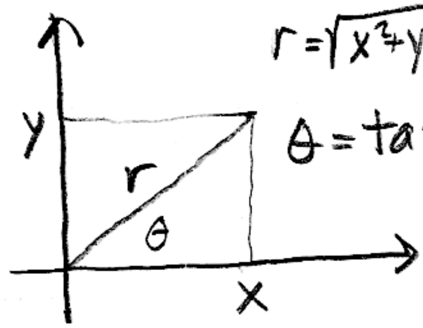


lines of constant x } $\hat{j}$ indep. of location (increasing y)



lines of constant y } $\hat{i}$ indep of location (increasing x)

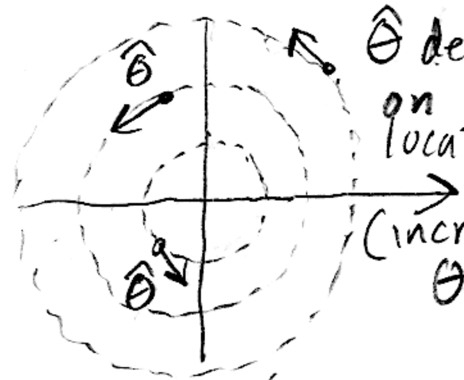
"Polar" r-θ



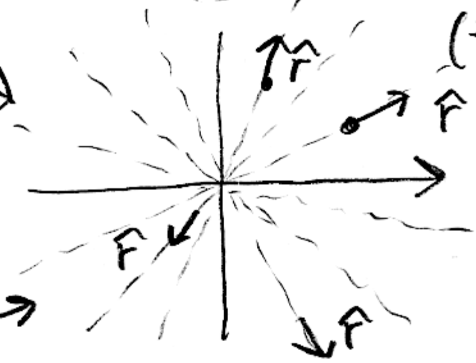
$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

lines of constant r



$\hat{\theta}$ depends on location! (increasing θ)



(toward $\hat{r}$ increasing r)

lines of constant θ

work in first quadrant

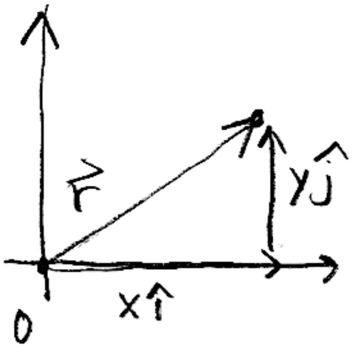
$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\hat{r}\cdot\hat{r} = \hat{\theta}\cdot\hat{\theta} = 1$$

$$\hat{\theta}\cdot\hat{r} = \hat{r}\cdot\hat{\theta} = (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\cdot(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= -\cos\theta\sin\theta\,\hat{i}\cdot\hat{i} + \cos^2\theta\,\underbrace{\hat{i}\cdot\hat{j}}_{0} - \sin^2\theta\,\underbrace{\hat{j}\cdot\hat{i}}_{0} + \sin\theta\cos\theta\,\hat{j}\cdot\hat{j} = 0$$



$\vec r = x\hat\imath + y\hat\jmath$ $r \equiv |\vec r|$

$x = r\cos\theta$ $y = r\sin\theta$

$\vec r = r(\cos\theta\,\hat\imath + \sin\theta\,\hat\jmath)$

$\vec r = r\hat r$ θ buried in $\hat r$...

$\boxed{\text{POSITION NOW MATTERS...}}$

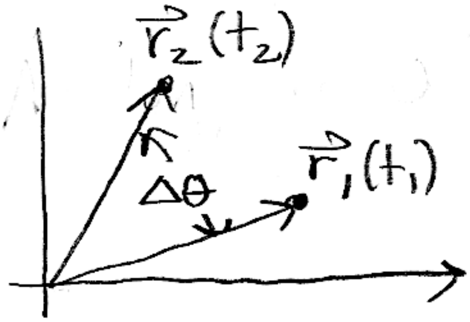
<u>Velocity</u> .. gets interesting

$$\vec v = \frac{d\vec r}{dt} = \frac{d}{dt}(r\hat r) = \underbrace{\frac{dr}{dt}\hat r}_{\text{old}} + \underbrace{r\frac{d\hat r}{dt}}_{\text{new}}$$

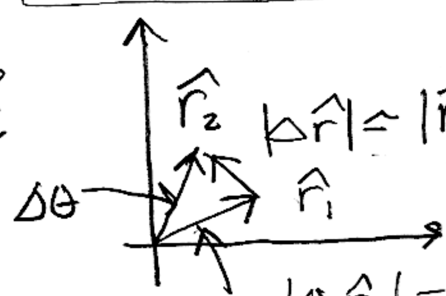
because.. $\frac{d\hat\imath}{dt} = \frac{d\hat\jmath}{dt} = 0$, $\hat\imath, \hat\jmath$ independent of position.

how can $\frac{d\hat r}{dt} \neq 0$?

1. ~~Length could change~~ → 0 unit v.
2. $\boxed{\text{direction could change}}$



$\hat r$?



$|\Delta\hat r| \simeq |\hat r|\Delta\theta$

$$\frac{|\Delta\hat r|}{\Delta t} = \frac{\Delta\theta}{\Delta t} \Rightarrow \dot\theta$$

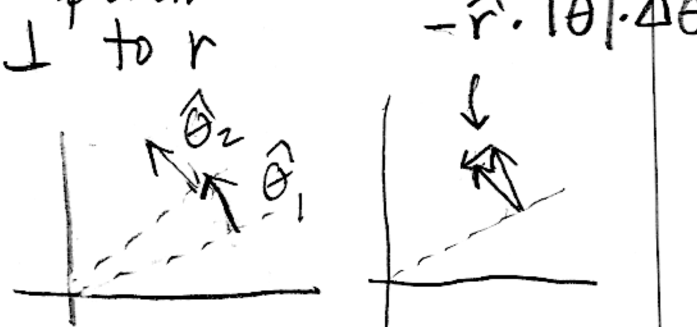
direction of $\Delta\hat r$, as $\Delta\theta \to 0$ .. $\underbrace{\perp \text{ to } \hat r_1}_{\hat\theta}$.

commonly,

$\dot\theta = \text{constant} = \omega$ $\frac{\text{radians}}{\text{second}}$ or $\boxed{\frac{d\hat r}{dt} = \dot\theta\,\hat\theta}$

so $\boxed{\dfrac{d\vec r}{dt} = \underbrace{\dfrac{dr}{dt}\hat r}_{\substack{\text{velocity}\\ \text{component}\\ \parallel \text{ to } \vec r}} + \underbrace{r\dot\theta\hat\theta}_{\substack{\text{velocity}\\ \text{component}\\ \perp \text{ to } r}}}$

Oh yes! $\dfrac{d\hat\theta}{dt} \longrightarrow$

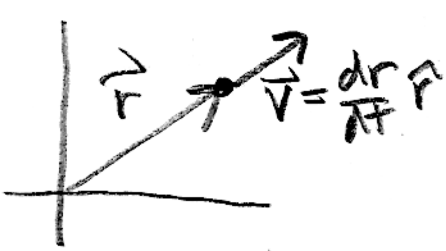


$\Delta\hat\theta = -\hat r\,\Delta\theta$

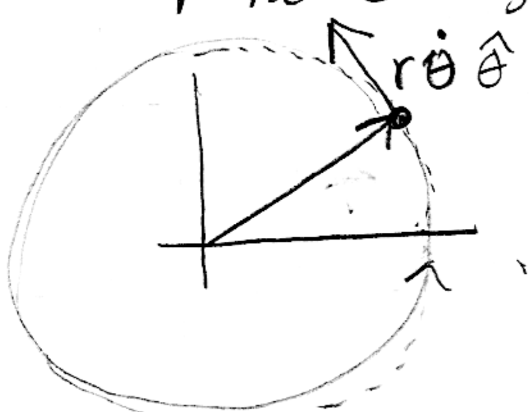
$$\boxed{\frac{d\hat\theta}{dt} = -\hat r\,\dot\theta}$$

$$\frac{d\vec r}{dt} = \frac{dr}{dt}\hat r + \underbrace{r\dot\theta\hat\theta}_{0} \qquad 0 \qquad r\dot\theta\hat\theta$$

$\theta \to$ no change $\qquad\qquad$ r no change



"along a spoke"



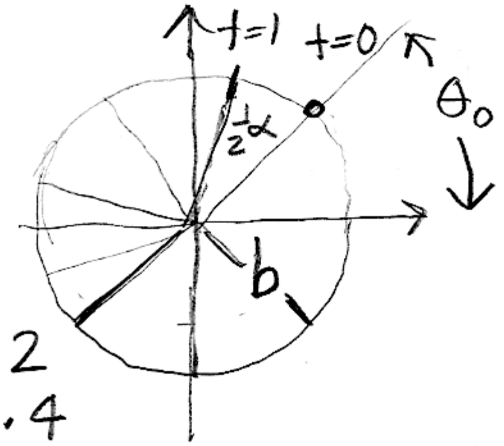
"around + around"

Ex 1.13 $\qquad r = b =$ constant

$$\dot\theta = \alpha t = \frac{d\theta}{dt}$$

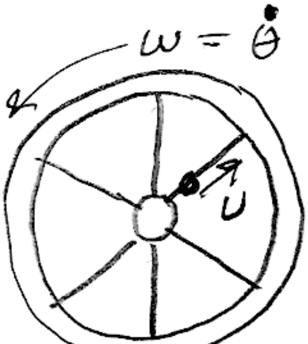
$$\frac{d\theta}{dt} = \alpha t \quad \text{Then} \quad \theta = \tfrac{1}{2}\alpha t^2 + \theta_0$$

when $t=0$
$\theta = \theta_0$



$t = 2$
$\frac{1}{2}\alpha \cdot 4$

Bead on Spoke



(a)

$$\vec{v} = \upsilon\hat{r} + (r_0 + \upsilon t)\cdot\omega\cdot\hat{\theta}$$

$\frac{dr}{dt} = \upsilon$ $\qquad \frac{d\theta}{dt} = \omega \qquad \theta = \omega t + \theta_0$

$r = \upsilon t + r_0$

$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

Cartesians (b) $\vec{v} = \upsilon\hat{r} + (r_0 + \upsilon t)\,\omega\,\hat{\theta}$

$$= \upsilon\left[\cos(\omega t + \theta_0)\hat{\imath} + \sin(\omega t + \theta_0)\hat{\jmath}\right]$$
$$+ (r_0 + \upsilon t)\,\omega\left[-\sin(\omega t + \theta_0)\hat{\imath} + \cos(\omega t + \theta_0)\hat{\jmath}\right]$$

$$= \left[\upsilon\cos(\omega t + \theta_0) - (r_0 + \upsilon t)\,\omega\sin(\omega t + \theta_0)\right]\hat{\imath}$$
$$+ \left[\upsilon\sin(\omega t + \theta_0) + (r_0 + \upsilon t)\,\omega\cos(\omega t + \theta_0)\right]\hat{\jmath}$$

messy!