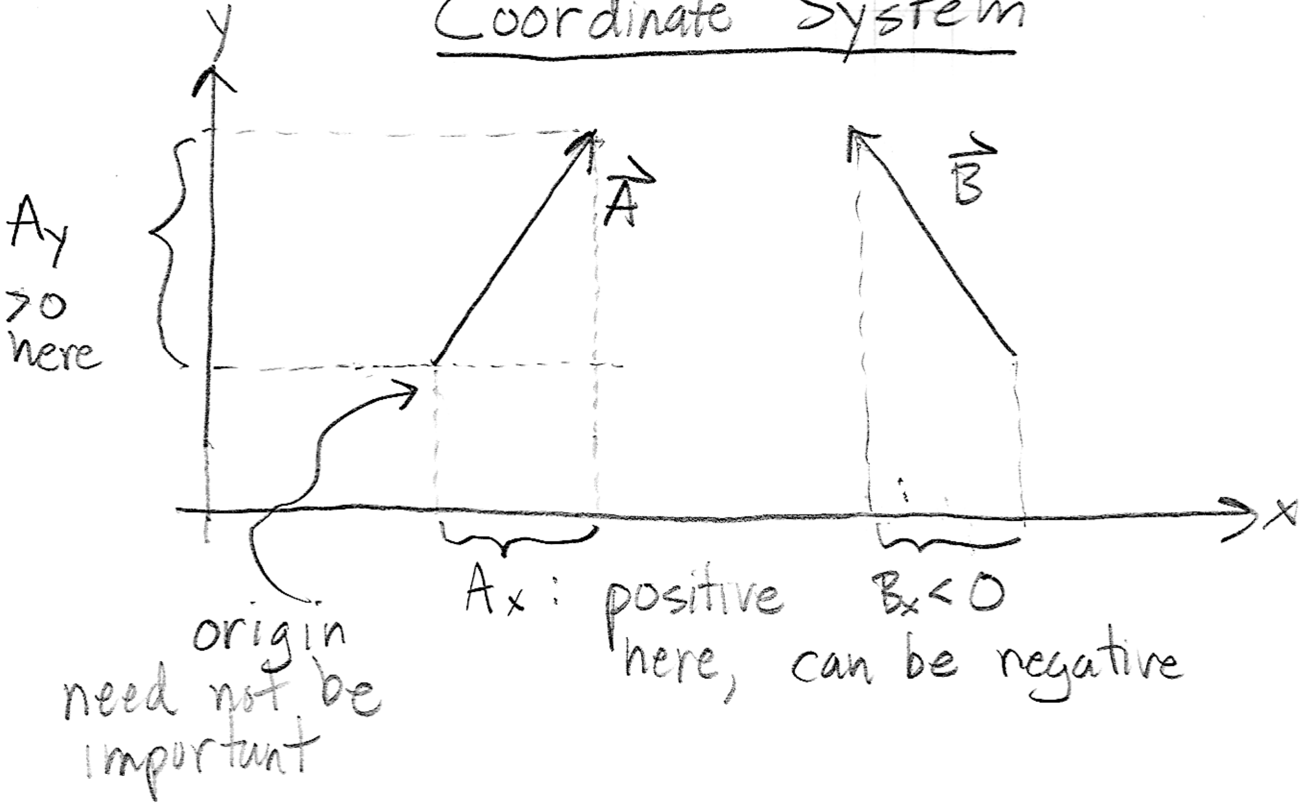
The Components of a Vector

vectors → "abstract" concept

dealing with them is facillitated by getting concrete, but, to due so, must choose a

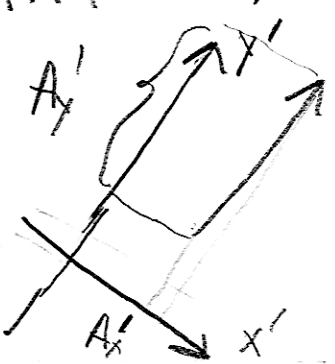
Coordinate System



Values A_x + A_y depend on both the nature of $\vec{A}$ and on the coordinate system. Kind of annoying. However, the magnitude of $\vec{A}$, $|\vec{A}|$, by the Pythagorean Theorem is:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

← this quantity is independent of the coordinate system



$$\sqrt{A_x'^2 + A_y'^2} = \sqrt{A_x^2 + A_y^2} = |\vec{A}|$$

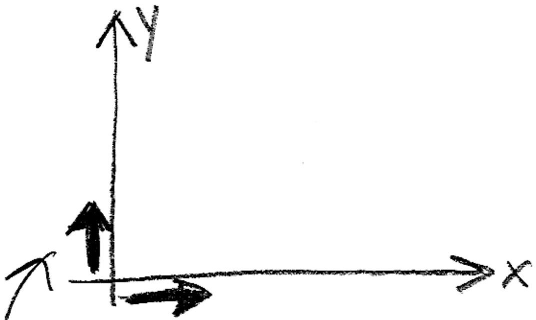
but $A_x' \neq A_x$ $\quad A_y \neq A_y'$

One way we write the component relation is:

$$\vec{A} = \underbrace{(A_x, A_y)}$$

not really the vector, really its components.

To get an equality with vectors on <u>both</u> sides of the equation we introduce the concept of <u>base</u> vectors



unit vector in x-direction called $\hat{\imath}$; $\hat{\imath}\cdot\hat{\imath} = |\hat{\imath}|^2 = 1$

unit vector in y-direction called $\hat{\jmath}$, $\hat{\jmath}\cdot\hat{\jmath} = |\hat{\jmath}|^2 = 1$

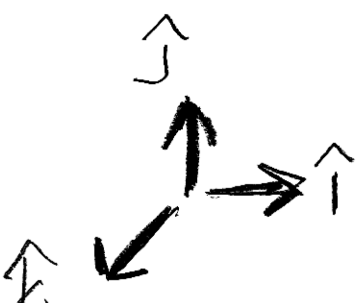
$$\hat{\imath}\cdot\hat{\jmath} = |\hat{\imath}||\hat{\jmath}|\underbrace{\cos\theta}_{0!} = 0$$

$\hat{\imath}\times\hat{\jmath} =$ 1) magnitude

$|\hat{\imath}||\hat{\jmath}|\sin(\theta = 90^\circ = \pi/2)$

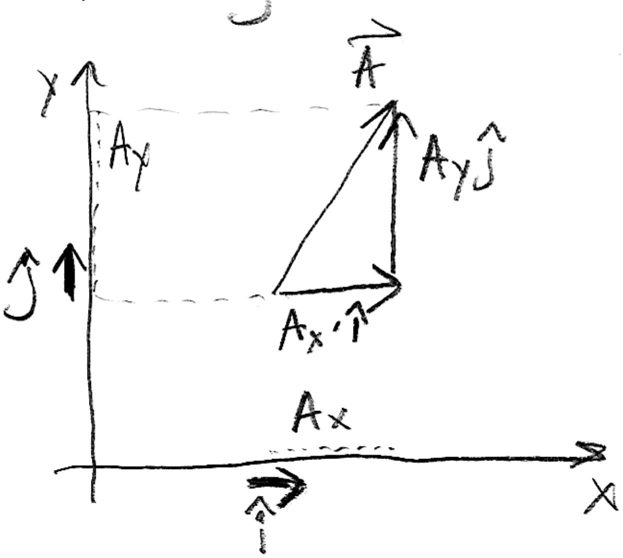
$= 1$

2) direction : right hand rule



$\Rightarrow$ z axis, $\Rightarrow \hat{k}$

What good are these?



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

both sides a vector.

note: $\vec{A}\cdot\vec{A} = (A_x\hat{i} + A_y\hat{j})\cdot(A_x\hat{i} + A_y\hat{j})$

$$= A_x^2 \underbrace{\cancel{\hat{i}\cdot\hat{i}}}_{1} + A_xA_y \underbrace{\cancel{\hat{i}\cdot\hat{j}}}_{0} + A_yA_x \underbrace{\cancel{\hat{j}\cdot\hat{i}}}_{0} + A_y^2 \underbrace{\cancel{\hat{j}\cdot\hat{j}}}_{1}$$

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 = |\vec{A}|^2$$

even more interesting

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j})(B_x\hat{i} + B_y\hat{j})$$

$$= A_xB_x\underbrace{\hat{i}\cdot\hat{i}}_{1} + A_xB_y\underbrace{\cancel{\hat{i}\cdot\hat{j}}}_{0} + \underbrace{\cancel{A_yB_x\hat{j}\cdot\hat{i}}}_{0} + A_yB_y\underbrace{\hat{j}\cdot\hat{j}}_{1}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$$

$$= |\vec{A}||\vec{B}|\cos\theta$$

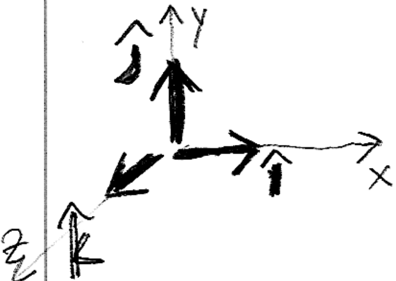
$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$|\vec{B}| = \sqrt{B_x^2 + B_y^2}$

$$\boxed{\cos\theta = \frac{A_xB_x + A_yB_y}{\sqrt{A_x^2 + A_y^2}\cdot\sqrt{B_x^2 + B_y^2}}}$$

θ is angle between $\vec{A}$ + $\vec{B}$

Cross Product with Base Vectors



$\hat{\imath}\times\hat{\jmath}=\hat{k}$ and $\hat{\jmath}\times\hat{\imath}=-\hat{k}$

Permute Cyclically!

$\hat{\jmath}\times\hat{k}=\hat{\imath}$ and $\hat{\imath}\times\hat{k}=-\hat{\jmath}$

$\hat{k}\times\hat{\imath}=\hat{\jmath}$ $\qquad$ $\hat{k}\times\hat{\jmath}=-\hat{\imath}$

$$\hat{\imath}\times\hat{\imath}=\hat{\jmath}\times\hat{\jmath}=\hat{k}\times\hat{k}=0$$

$$(A_x\hat{\imath}+A_y\hat{\jmath})\times(B_x\hat{\imath}+B_y\hat{\jmath})$$

$$=A_xB_x\underbrace{(\hat{\imath}\times\hat{\imath})}_{0}+A_xB_y\underbrace{(\hat{\imath}\times\hat{\jmath})}_{\hat{k}}+A_yB_x\underbrace{(\hat{\jmath}\times\hat{\imath})}_{-\hat{k}}+A_yB_y\underbrace{(\hat{\jmath}\times\hat{\jmath})}_{0}$$

$$\vec{A}\times\vec{B}=(\underbrace{A_xB_y-A_yB_x}_{\text{resembles determinant of}})\hat{k}$$

$$\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

In $\underset{\sim}{3}$ dimensions,

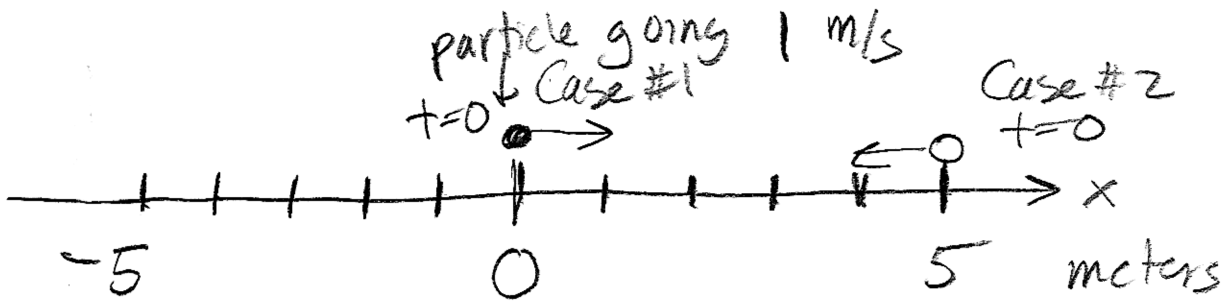
$$\vec{A}\times\vec{B}=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

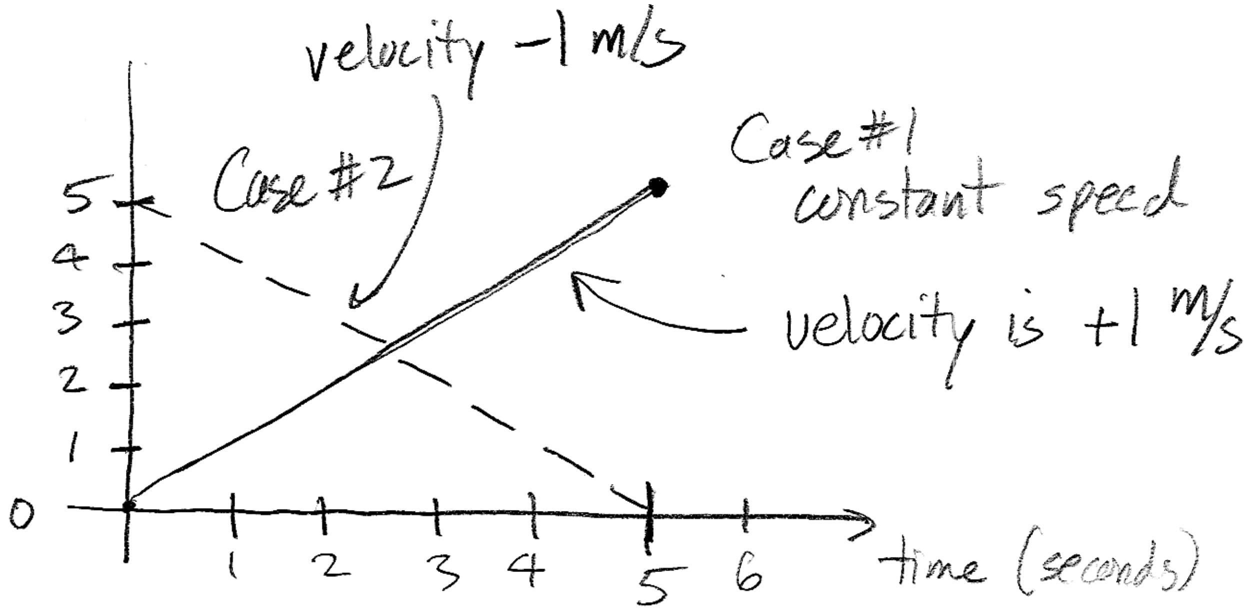
Displacement, Velocity, Acceleration

Move from position $\vec{r}_1$ to $\vec{r}_2$

position vector: from coordinate system
origin: (0) or (0,0) or (0,0,0)
to (x_1) (x_1, y_1) or to (x_1, y_1, z_1)
... Depends on where origin placed!

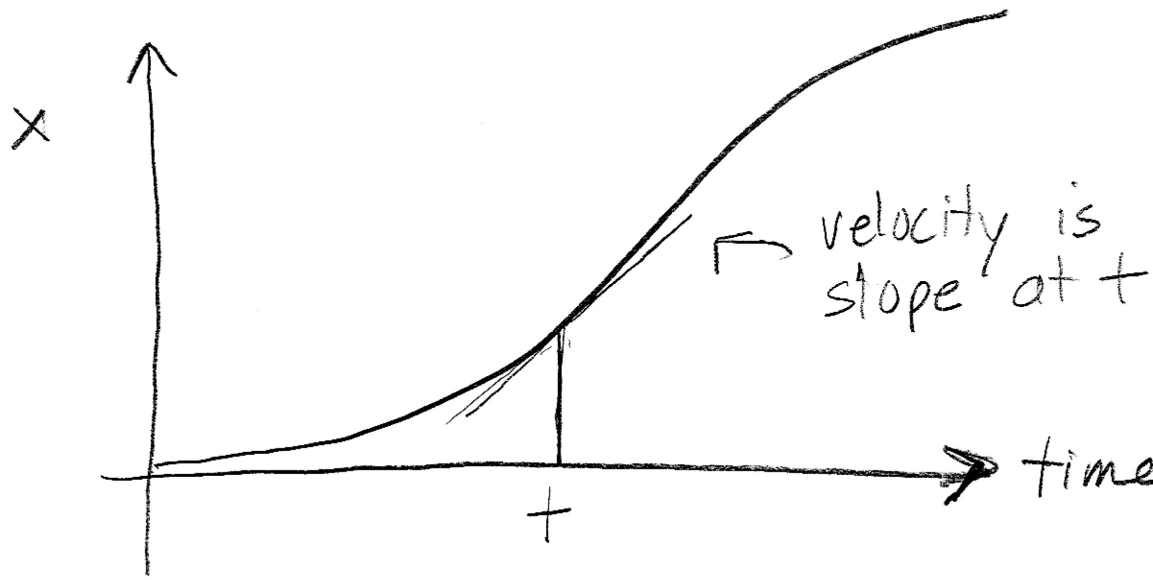
Displacement is $\vec{r}_2 - \vec{r}_1$ ... origin cancels.
"start to finish"





velocity incorporates direction
velocity is slope

Maybe velocity not constant!
(foot on the gas!)
$\Rightarrow$ acceleration $\neq 0$



slope at t is: $\frac{dx}{dt}$ or x' or $\dot{x}$

in more dimensions...

<u>Position</u>: $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dx(t)}{dt}\hat{\imath} + \frac{dy(t)}{dt}\hat{\jmath} + \frac{dz(t)}{dt}\hat{k}$$

These do not change in time, so their derivatives are zero

$$v_x(t) = \frac{dx}{dt}(t)$$

$$v_y(t) = \frac{dy}{dt}(t)$$

$$v_z(t) = \frac{dz}{dt}(t)$$

$$\vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} + v_z(t)\hat{k}$$

$$\text{speed} = |\vec{v}(t)| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}$$