Vectors

- new mathematical object... more complicated than "real numbers" ⇒ call "scalars", usually
- invented for economical description of physical situations
- vectors are an abstraction... there is something deeper present than just the "engineering" of working with them

Vector ⟺ something with magnitude (real # ≥ 0) and direction (usually in our physical space, 1, 2, 3 d; sometimes more "abstract" space)

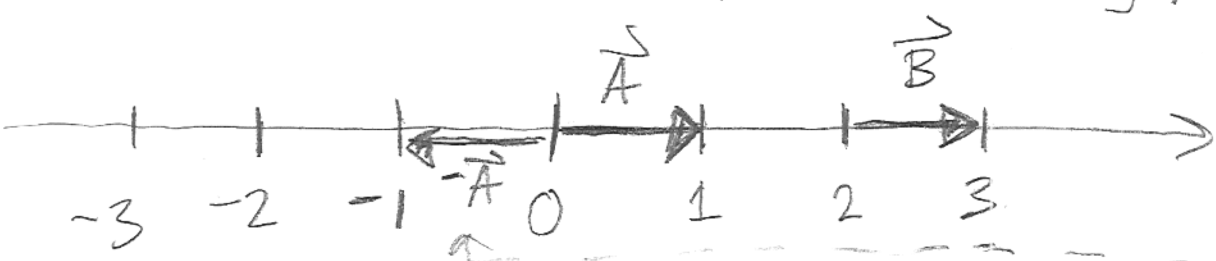
need not have "location," although it might!

notation: $\vec{A}$ ← arrow on top (hand written)

$\mathbf{A}$ boldface (type set)

magnitude $|\vec{A}|$ or $|\mathbf{A}|$

1-dimension: $\vec{A}$: say, from 0 to 1; $\vec{B}$: say, from 2 to 3 } location not import. $\vec{A} = \vec{B}$



There are actually 2 directions in 1-d;
Use − sign to specify other direction...
Or, multiplication by the scalar −1 "flips direction"

More generally, multiplication of a vector by a scalar (not necessarily ± 1):

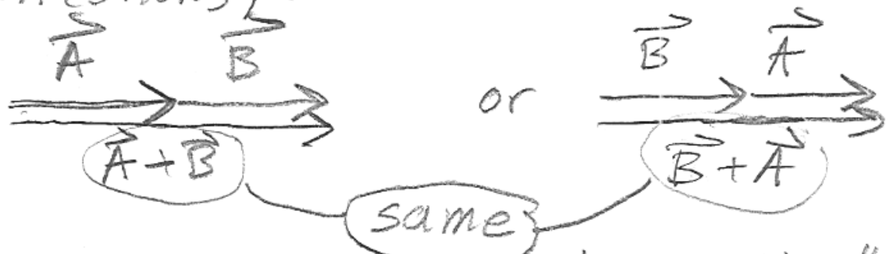
- flips direction of the vector if the scalar is < 0
- changes the magnitude from, say,

$$|\vec{A}| \text{ to } |c| \cdot |\vec{A}|$$

c is the scalar

Addition of Vectors

Should be $\vec{A} + \vec{B}$ has length 2. (Referring to 1-d example on p.1). Since location unimportant, can translate the vectors around, but to get something of length 2, better make them "tip to tail"! (Can <u>not</u> flip their directions):



All these operations behave "normally," that is, they commute, distribute, etc. just like real numbers (p. 4 of text).

Multiplication of Vectors by Vectors?

1-d ... not very much fun

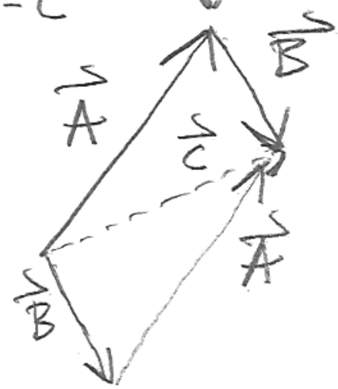
2, 3, 4, ... dimensions ... THAT IS FUN

but COMPLICATED because <u>MANY TYPES</u> of multiplication

Into the SECOND dimension...

still, $\vec{A} = \vec{D}$; location unimportant (assuming parallel, same magnitude)

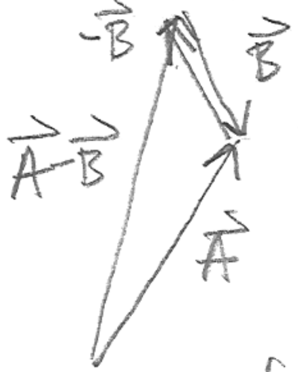
$\vec{A} + \vec{B} \equiv \vec{C}$ tip to tail



or, $\vec{B} + \vec{A}$

- add by tip to tail
- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- 3-d, 4-d, etc... concept same

$\boxed{\text{Subtraction}}$ $\vec{A} - \vec{B} = \vec{A} + \underbrace{(-\vec{B})}_{\text{add the flipped}}$



put $\vec{A}$ + $\vec{B}$ "head to head" the missing side is $\vec{A} - \vec{B}$, as long as start at base of $\vec{A}$ and proceed to base of $\vec{B}$

also,

$\vec{A} - \vec{B}$ → can put base-to-base and $\vec{A} - \vec{B}$ will be missing side, head-to-head with $\vec{A}$, the first vector in the difference

Vector Multiplication with other Vector

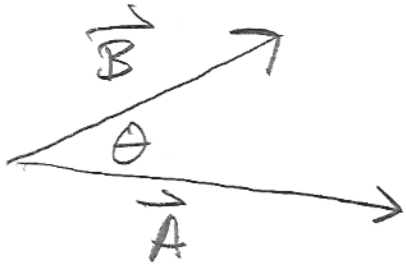
① The "Scalar" or "Dot" product
out come is a <u>scalar</u>.

② The "Vector" or "Cross" prodot
out come is a <u>vector</u>

③ not discussed here... "higher" products,
learn in Quantum, 115B/C.

Scalar Product

- Put $\vec{A}$ + $\vec{B}$ tail to tail (will always be in a plane)



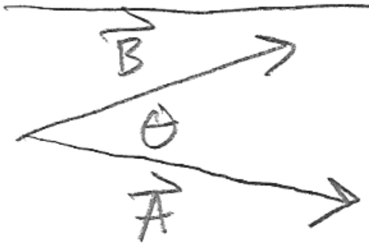
- Label angle between them (short way; direction doesn't matter) θ
- $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = |\vec{A}||\vec{B}|\cos(-\theta)$ } $\vec{A}\cdot\vec{A} = A^2$
- can interpret as one projected on other, or, vice versa

Question:

(other questions too)

$\vec{A}\cdot\vec{B}$: > 0, $= 0$, < 0 ?

Vector Product



answer $\propto \sin\theta$ this time!
note: $\sin\theta \neq \sin(-\theta)$!!
direction of θ matters!

$\vec{A}\times\vec{B}$: take your <u>right</u> hand.
"assume the position" → thumb sticking up
curled fingers
("hitchhiker position")

4 fingers: from $\vec{A}$ to $\vec{B}$
THUMB parallel to $\vec{A}\times\vec{B}$ (direction)
magnitude: $|\vec{A}||\vec{B}|\sin\theta$

$\vec{B}\times\vec{A}$: must flip hand to do it ...
in opposite direction from
$\vec{A}\times\vec{B}$!

so, $\vec{B}\times\vec{A} = -(\vec{A}\times\vec{B})$

note:

same

$$(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$$
$$= A^2 + 2\vec{A}\cdot\vec{B} + B^2 \quad \text{(law of cosines)}$$

$$\underbrace{(\vec{A}+\vec{B})\times(\vec{A}+\vec{B})}_{\text{know zero, because no angle between}} = \underbrace{\cancel{\vec{A}\times\vec{A}}}_{0} + \underbrace{\vec{A}\times\vec{B} + \vec{B}\times\vec{A}}_{\text{add to zero.}} + \underbrace{\cancel{\vec{B}\times\vec{B}}}_{0}$$

$\boxed{\Rightarrow \text{vector product doesn't commute!}}$

Unit Vector

arbitrary non-zero vector $\vec{A}$

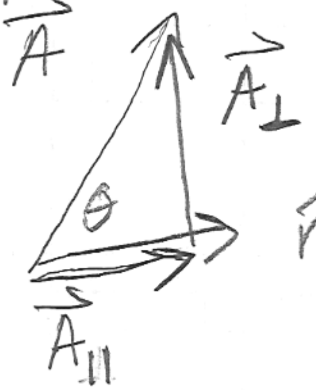
consider $\left(\frac{1}{A}\cdot\vec{A}\right) \equiv \hat{A}$

$$\hat{A}\cdot\hat{A} = |\hat{A}|^2 = \frac{1}{A^2}\cdot|\vec{A}|^2 = 1$$

vector with "unit norm" called a unit vector. Has direction, but no interesting magnitude. (Complements scalars)

Para-Perp Decomposition

arbitrary vector $\rightarrow \vec{A}$



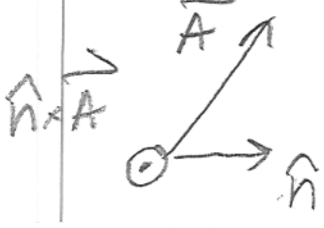
$\hat{n}$ = unit vector in some cool direction

$\vec{A} = \vec{A}_\parallel + \vec{A}_\perp \leftarrow$ $\vec{A}_\parallel$ parallel (antiparallel) to $\hat{n}$

$\vec{A}_\perp$ perpendicular to $\hat{n}$

$\vec{A}_\parallel = (\hat{n}\cdot\vec{A})\cdot\hat{n}$ pretty easy

$\vec{A}_\perp$: $\hat{n}\times\vec{A}$ has correct magnitude $(|\vec{A}||\sin\theta|)$ wrong direction



$(\hat{n}\times\vec{A})\times\hat{n} = \vec{A}_\perp$, since

$(\hat{n}\times\vec{A})$ + $\hat{n}$ are at $\perp$ angle.