

Vectors in Mathematica

K&K 1.2: $\vec{A} = 3\hat{i} + \hat{j} + \hat{k} = (3, 1, 1)$ $\vec{B} = -2\hat{i} - 3\hat{j} - \hat{k} = (-2, -3, -1)$

Mathematica $A = \{3, 1, 1\}$; $B = \{-2, -3, -1\}$; $\boxed{S-E}$

\swarrow means "quiet"

Dot-Product $\rightarrow \bullet$

$$A \cdot B \quad \boxed{S-E}$$

$$-10$$

Magnitude of a vector: Norm $[A]$

$$\text{Norm}[A] \quad \boxed{S-E}$$

$$\sqrt{||}$$

$$\cos \theta_{AB} = N [A \cdot B / \text{Norm}[A] / \text{Norm}[B]]$$

$$= -0.805823$$

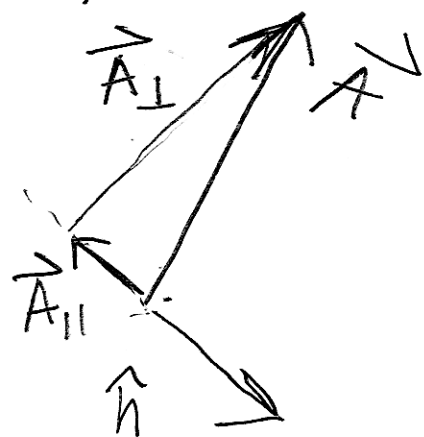
Cross-Product: Cross $[A, B]$

K&K 1.11

\hat{n} = some unit vector
 \Rightarrow interesting direction
 in a problem.

Mathematica: $n = \{5, -6, 1/2\}$;

$$\hat{n} = n / \text{Norm}[n] \quad \boxed{S-E}$$



$$\vec{A}_{||} = (\vec{A} \cdot \hat{n}) \hat{n}$$

Mathematica: $A_{pa} = (\vec{A} \cdot \hat{n}) \hat{n}$ [S-E]

$$(0.77551, -0.930612, 0.077551)$$

$$\hat{n} \cdot A_{pa} / \text{Norm}[\hat{n}] / \text{Norm}[A_{pa}]$$
 [S-E]

1.

could have returned -1. too, for other cases.

$$\vec{A}_{\perp} = (\hat{n} \times \vec{A}) \times \hat{n}$$

Mathematica: $A_{pe} = \text{Cross}[\text{Cross}[\hat{n}, \vec{A}], \hat{n}]$ [S-E]

$$(2.22449, 1.93061, 0.922449)$$

$$A_{pa} + A_{pe}$$
 [S-E]

$$\{3., 1., 1.\}$$
 (same as \vec{A})

$$A_{pa} \cdot A_{pe}$$
 [S-E]

$$-8.32667 \cdot 10^{-17}$$

$$\hat{n} \cdot A_{pe} / \text{Norm}[\hat{n}] / \text{Norm}[A_{pe}]$$
 [S-E]

$$-4.72109 \cdot 10^{-17}$$

Parametric Plots

$$\vec{r}(t) = (x(t), y(t))$$

$$= x(t)\hat{i} + y(t)\hat{j}$$

simple case: $x(t) = x_0 + v_x t$

$$y(t) = y_0 + v_y t$$

mm: take $x_0 = 1\text{m}$ $v_x = 2\text{ m/s}$

$$y_0 = -1\text{m}$$

$$v_y = -4\text{ m/s}$$

could plot 2 "normal" plots

$$\text{Plot}[1+2*t, \{t, 0, 5\}] \quad \boxed{\text{S-E}}$$

$$\text{Plot}[-1-4*t, \{t, 0, 5\}] \quad \boxed{\text{B-E}}$$

Or, Parametric Plot does both x & y as a function of time.

$$\text{Parametric Plot}[\{1+2*t, -1-4*t\}, \{t, 0, 5\}]$$

Set Options [Plot
ParametricPlot, TextStyle \rightarrow
{FontSize \rightarrow 18}]

or

$$x[t_] := 1 + 2*t$$

$$y[t_] := -1 - 4*t$$

$$\text{Parametric Plot}[\{x[t], y[t]\}, \{t, 0, 5\}]$$

Velocity and Speed

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt} \right) = (\dot{x}, \dot{y})$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} = \dot{x} \hat{i} + \dot{y} \hat{j}$$

\uparrow \uparrow \uparrow \uparrow
 $v_x(t)$ $v_y(t)$ $v_x(t)$ $v_y(t)$

$$\vec{v} = (v_x(t), v_y(t)) = v_x(t) \hat{i} + v_y(t) \hat{j}$$

Mathematica:

$$\begin{aligned} & D[x[t], t] \quad [S-E] \\ & -2 \\ & D[y[t], t] \quad [S-E] \\ & -4 \end{aligned}$$

$$\vec{v} = (2, -4) \text{ m/s}$$

$$v_x[t] := D[x[t], t]$$

$$v_y[t] := D[y[t], t]$$

Parametric Plot $\{v_x[t], v_y[t]\}, \{t, 0, 5\}$

$$\text{Speed} = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Repeat / Replace with

$$x(t) = \left(\frac{t}{2\pi}\right) \cos [2\pi t]$$

$$y(t) = -\left(\frac{t}{2\pi}\right) \sin [2\pi t]$$