Translation	Rotation
$\vec{x}$	θ (not a vector!)
$\vec{v} = \dot{\vec{x}}$	$\vec{\omega}$
$\vec{a} = \dot{\vec{v}}$	$\vec{\alpha} = \dot{\vec{\omega}}$ $[1/s^2]$
m	I $[m\,l^2]$
$\vec{F}$	Torque $\vec{\tau}$ $[\,?\,]$

$$\vec{F}_{Net} = m\vec{a} \xrightarrow{?} \vec{\tau}_{Net} = I\vec{\alpha}$$

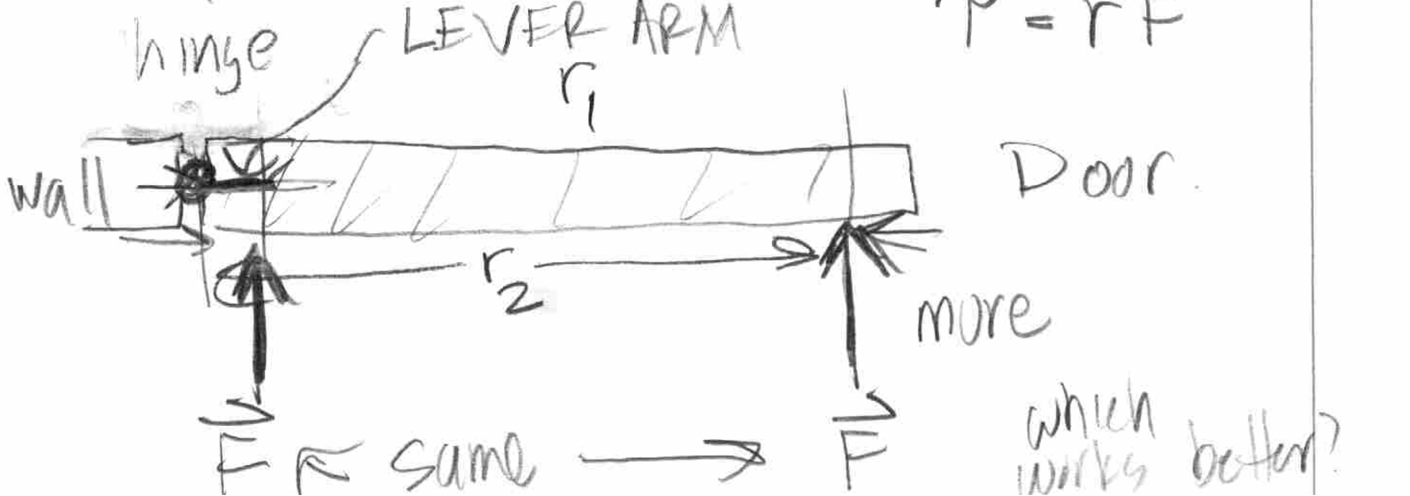
$$\left[\frac{m \cdot l}{s^2}\right] \qquad [I\vec{\alpha}] = \frac{m\,l^2}{s^2}$$

$$[\vec{\tau}] \rightarrow l \cdot [\text{Force}]$$

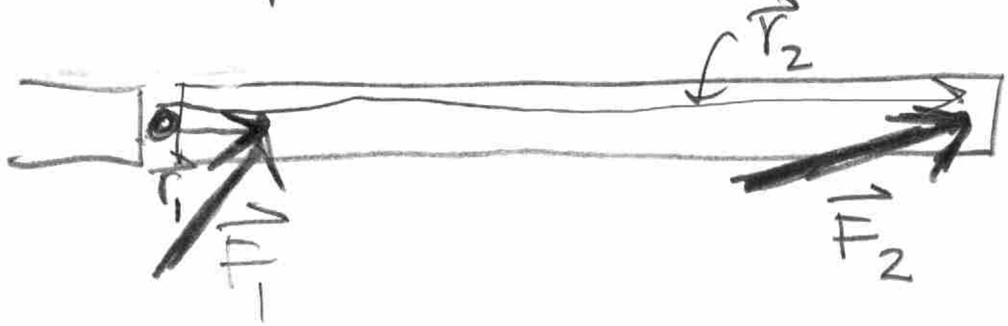
!!

Torque incorporates concept of force and lever arm here...

$\tau = rF$



Suppose you were silly enough not to push $\perp$ to door...



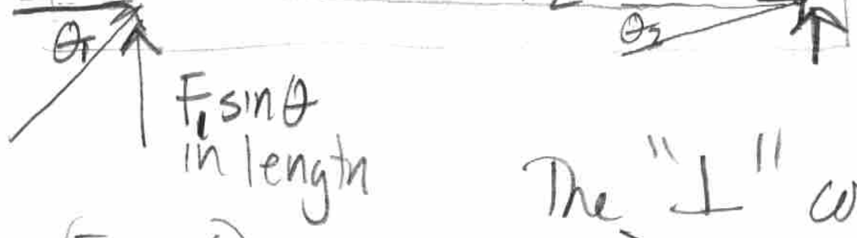
Cross Product Captures the reduction of effectiveness! More generally

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}|\,|\vec{F}| \sin\theta = rF\sin\theta$$

CONCEPTUALLY, $\sin\theta$ can go with either the r or the F!

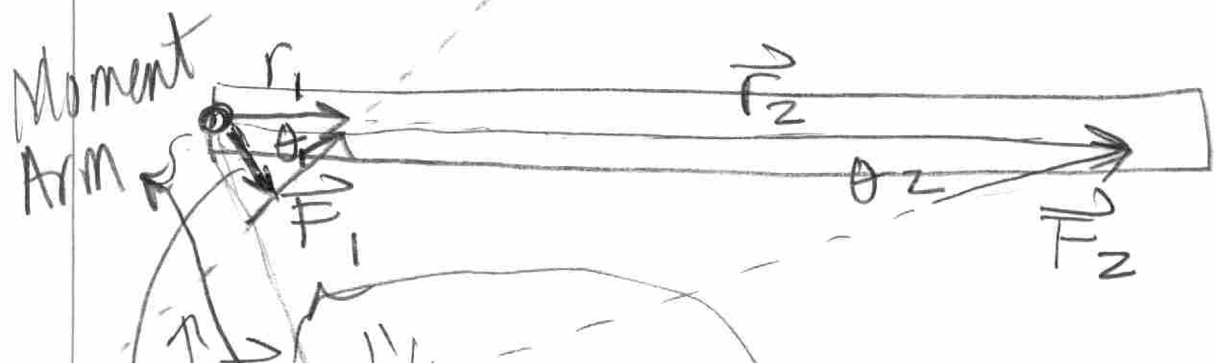
Above .. with F



$\tau_2 = r_2 F_{2\perp}$

$\tau_1 = r_1 (F_1 \sin\theta)$
$= r_1 F_{1\perp}$

The "$\perp$" component of $\vec{F}$, times the radius OR



$\vec{F}$'s "line of action"

$r_{2\perp} = r_2 \sin\theta_2$

$r_{1\perp} = r_1 \sin\theta$

$\tau_1 = (r_{1\perp}) F_1$

$= r_1 F_1 \sin\theta$

$= (r_{2\perp}) F_2$

$= r_2 F_2 \sin\theta_2$

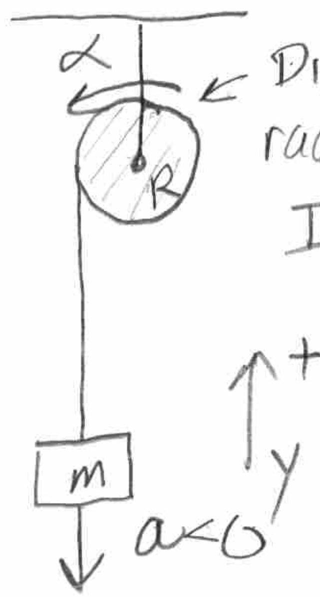
Simple rotation about an axis:

$\tau > 0$

$\tau < 0$

Generally reason out signs in specific cases .. careful!

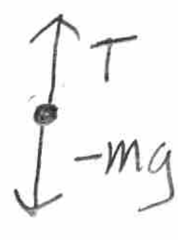
"1-d" $\tau_{Net} = I\alpha$ $\quad \alpha$ about the axis.



Disk, mass M, radius R, $I = \frac{1}{2}MR^2$

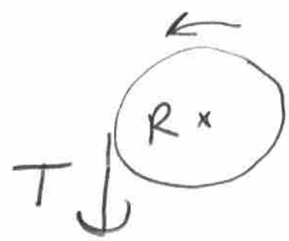
Find acceleration a angular acc. α

mass:



$$ma = T - mg$$

wheel:



$$\tau = TR = I\alpha = \frac{1}{2}MR^2\alpha$$

No slip: $a = -\alpha R$, $\alpha = \frac{-a}{R}$

so $T\cancel{R} = \frac{1}{2}MR^2\left(\frac{-a}{R}\right) = -\frac{1}{2}M\cancel{R}a$

$$T = -\frac{1}{2}Ma$$

plug in: $ma = -\frac{1}{2}Ma - mg$

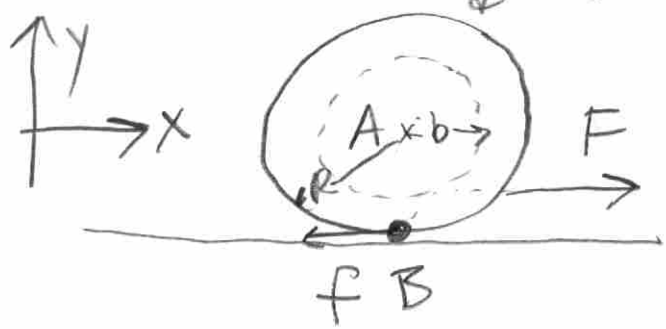
$$mg = \left(-\frac{1}{2}M - m\right)a$$

$$\boxed{a = -\frac{m}{\frac{1}{2}M + m}g = -\frac{2m}{M+2m}g}$$

$$\boxed{\alpha = -\frac{a}{R} = \frac{2m}{M+2m}\frac{g}{R}}$$

Pulling a Yo-Yo... M, I_{cm} Combined

Translational + Rotational

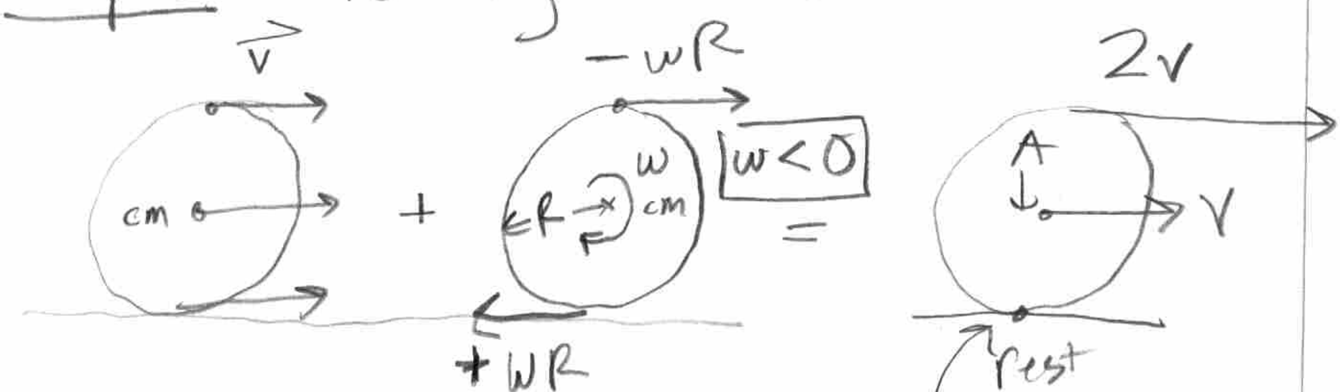


Forces... f & F : pulls, makes CENTER OF MASS translate.

F given

$F - f = Ma_x$

Torques rolling... look out



B rotation

$v + \omega R = 0$

$v = -\omega R, \; a_x = -\alpha R$

A : $I\alpha = +Fb - fR = -I\frac{a_x}{R}$

$f = F - Ma_x$

$Fb - (F - Ma_x)R = -\frac{I}{R}a_x$