What is <u>qualitatively</u> interesting?

(1) Straight-on = 1-d.

$$V_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} V_{1i}$$

$m_1 > m_2$, $V_{1f} > 0$

$m_1 < m_2$, $V_{1f} < 0$

(2) Not straight on --

(a) <u>TOTAL MISS</u> (not possible in 1-d)

$\phi_1 = \phi_2 = 0$

(b) HOW BIG CAN ϕ_2 GET?

→ 90° or $\phi_2 = \pi/2$

(c) HOW BIG CAN $|\phi_1|$ GET?

(i) $m_1 < m_2$ --- #1 can go <u>backward</u>

$0 < |\phi_1| < \pi$

(ii) $m_1 > m_2$ interesting...

$\sin |\phi_1|_{max} = \frac{m_2}{m_1}$ (!!)

THINK: • straight-on
+ TOTAL MISS
both give $\phi_1 = 0$
when $m_1 > m_2$
• Intermediate -- (ϕ_1 | gets big

Equations -- leave $\neq 0$ for now

momentum

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

2 components now:

x: along line between --
(assume $\vec{v}_{1i} + \vec{v}_{2i}$ collinear)

y: transverse.

x: $m_1 v_{1i} + m_2 v_{2i} = m_1|\vec{v}_{1f}|\cos\phi_1 + m_2|\vec{v}_{2f}|\cos\phi_2$

y: $0 = -m_1|\vec{v}_{1f}|\sin\phi_1 + m_1|\vec{v}_{2f}|\sin\phi_2$

energy

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1|\vec{v}_{1f}|^2 + \frac{1}{2}m_2|\vec{v}_{2f}|^2$$

4 unknowns: $|\vec{v}_{1f}|, |\vec{v}_{2f}|, \phi_1, \phi_2$

3 equations → 1 new independent variable.

possible: ϕ_1 or ϕ_2

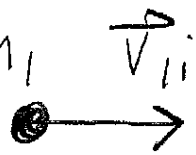
⇒ "natural", ⊖ : center of mass scattering angle.

First example of "frame hopping", crucial in relativity.

⇒ view collision in frame where total momentum is zero.

⇒ called "center of mass" frame

LAB FRAME



m_2 ($\vec{v}_{2i} = 0$ shown)

$\vec{V}$: velocity of center of mass

$$(m_1 + m_2)\vec{V} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$$

$$\vec{V} = \frac{m_1}{m_1+m_2}\vec{v}_{1i} + \frac{m_2}{m_1+m_2}\vec{v}_{2i}$$

LAB FRAME

m_2 ($\vec{V}_{2i} = 0$ shown)

$\rightarrow \vec{V}$

$\vec{V}_{1i}' = \vec{V}_{1i} - \vec{V}$ ← $\vec{V}_{1i}'$ in center of mass frame.

$= \left(1 - \frac{m_1}{m_1+m_2}\right)\vec{V}_{1i} - \frac{m_2}{m_1+m_2}\vec{V}_{2i}$

$\vec{V}_{1i}' = \frac{m_2}{m_1+m_2}\left(\vec{V}_{1i} - \vec{V}_{2i}\right)$

$\vec{V}_{2i}' = \vec{V}_{2i} - \vec{V}$

$= \left(1 - \frac{m_2}{m_1+m_2}\right)\vec{V}_{2i} - \frac{m_1}{m_1+m_2}\vec{V}_{1i}$

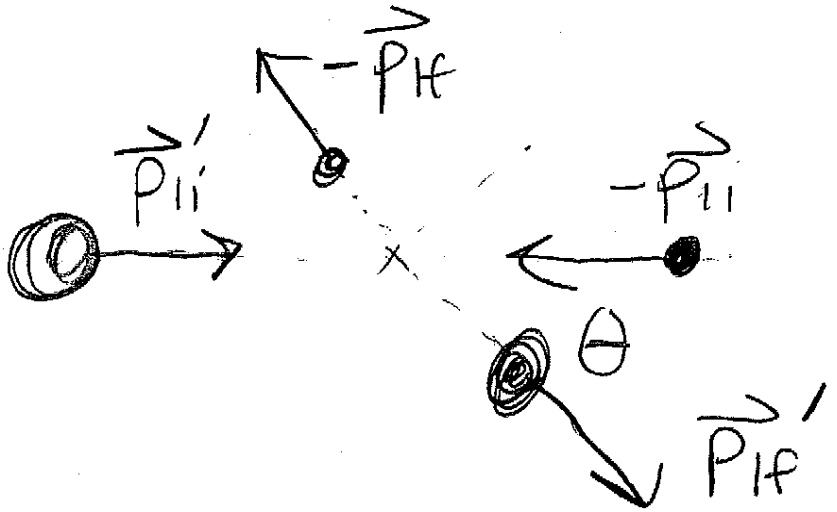
$\vec{V}_{2i}' = -\frac{m_1}{m_1+m_2}\left(\vec{V}_{1i} - \vec{V}_{2i}\right)$

From CM viewpoint...

$m_1 > m_2$ as shown

put $|m_1\vec{V}_{1i}'| = |m_2\vec{V}_{2i}'| = |\vec{P}_{1i}'| = |\vec{P}_{2i}'|$

Now, scatter.



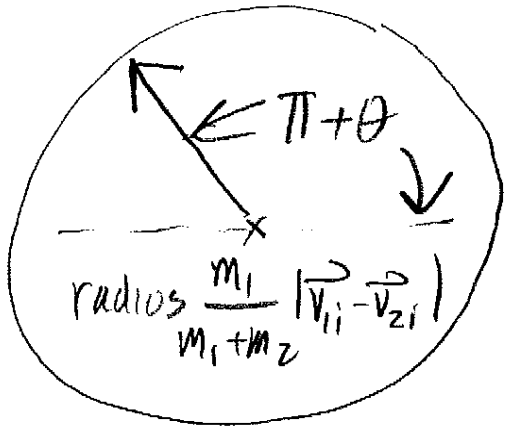
θ ... always $0 \to \pi$!

$$\vec{v}_{1f}' = \frac{\vec{p}_{1f}'}{m_1}$$

$$\vec{v}_{2f}' = \frac{-\vec{p}_{1f}}{m_2}$$

radius $\frac{m_2}{m_1+m_2}|\vec{v}_{1i}-\vec{v}_{2i}|$ — velocity "circles"

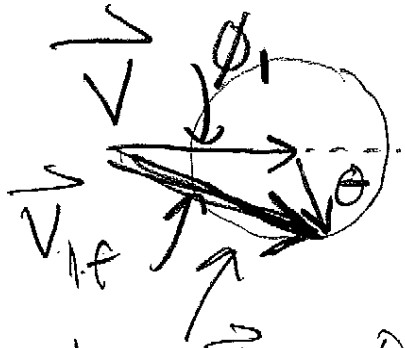
#1



#2

TO GET BACK TO LAB FRAME .. ADD BACK

Particle #1's Velocity Circle



when $\vec{V}_{2i} = 0$

radius

$$\frac{m_2}{m_1+m_2}|\vec{V}_{1i}|$$

$$< |\vec{V}| = \frac{m_1}{m_1+m_2}|\vec{V}_{1i}|$$

when $m_1 > m_2$

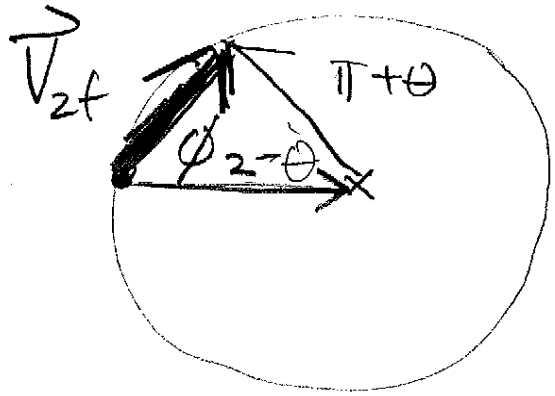
maximum $|\phi_1|$:

$$\frac{m_2}{m_1+m_1}|\vec{V}_{1i}|$$

$$\sin|\phi_{1\,max}| = \frac{\frac{m_2}{m_1+m_2}|\vec{V}_{1i}|}{\frac{m_1}{m_1+m_2}|\vec{V}_{1i}|}$$

$$= \frac{m_2}{m_1}$$

Particle #2's Velocity Circle



when $\vec{V}_{2i} = 0$

$$\text{radius} = \frac{m_1}{m_1+m_2}|\vec{V}_{1i}|$$

$$= |\vec{V}|$$

ϕ_2 max $\frac{\pi}{2}$