1-d collisions (Chapter 10 R+R4, p. 212)

given m_1 v_{1i} → given m_2 v_{2i} → Initial

$v_{1i} - v_{2i}$ →

(collision)

m_1 v_{1f} → v_{2f} →

$v_{2f} - v_{1f}$ →

(!)

Momentum: (if no external forces)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Energy: (if no "smushy", called ELASTIC)

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

get masses on same side

(A) $m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$

(B) $\frac{1}{2} m_1 (v_{1i}^2 - v_{1f}^2) = \frac{1}{2} m_2 (v_{2f}^2 - v_{2i}^2)$

Divide (B) by (A) pp. 188–194

(B) → $\frac{\frac{1}{2}m_1(v_{1i}+v_{1f})(v_{1i}-v_{1f})}{m_1(v_{1i}-v_{1f})} = \frac{\frac{1}{2}m_2(v_{2f}+v_{2i})(v_{2f}-v_{2i})}{m_2(v_{2f}-v_{2i})}$ ← (A)

$$v_{1i}+v_{1f} = v_{2f}+v_{2i}$$

or, $v_{1i}-v_{2i} = v_{2f}-v_{1f}$

Reasoning $m_1 = m_2 = m$ $v_{1i} = v$

$v_{2i} = 0$ $v_{2f}-v_{1f} = v-0 = v$

initial

$v_{1i}-v_{2i} = v-0$

$0 \quad = v$

"obviously" conserves E, p . . . right ??

$E = \frac{1}{2}mv^2$

$E = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2$

$= \frac{1}{2}m\left(\frac{1}{4}v^2 + \frac{1}{4}v^2\right)$

$= \frac{1}{4}mv^2$

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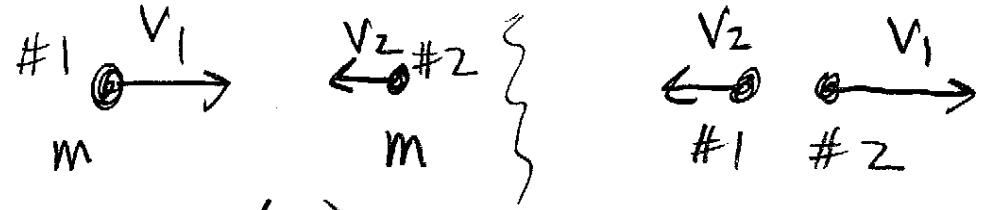


$V_{1i} - V_{2i} = V_1 - (-V_2)$

$= V_1 + V_2$

$V_{2f} - V_{1f} = V_1 - (-V_2)$

$= V_1 + V_2$

More fun... $m_1 >> m_2$

Freight Trane

elastic "Fly"

say, $V_{2i} = 0$

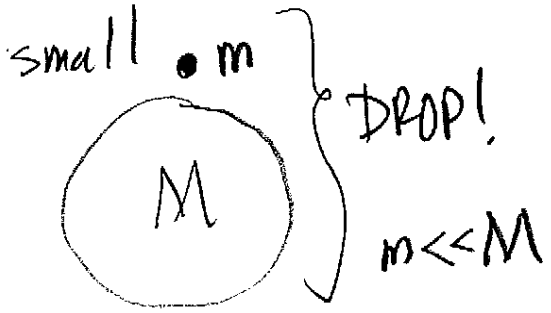
$V_{1i} - V_{2i} = V$

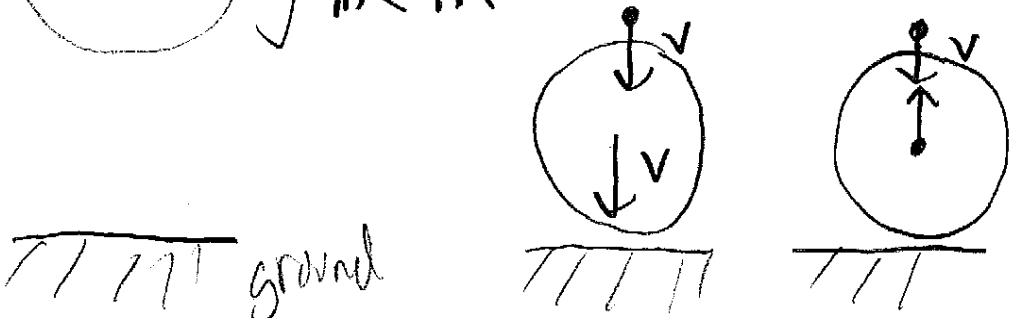
$V_{2f} - V_{1i} = V_{2f} - V = V$

$\boxed{V_{2f} = 2V}$

WOW!

Basketball Amplifier





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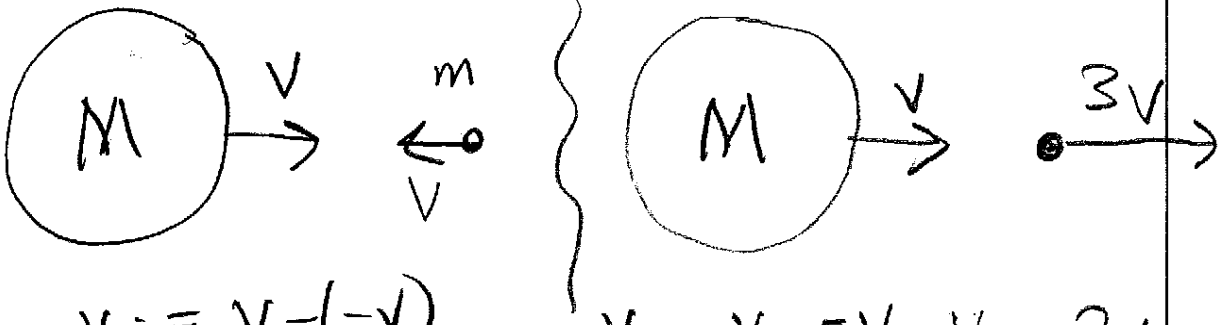
TURN SIDEWAYS

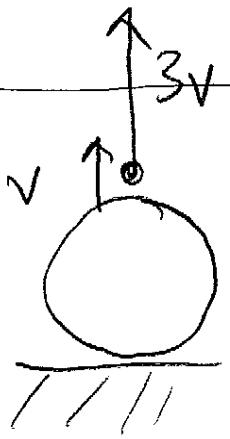


$v_{1i} - v_{2i} = v - (-v)$

$= 2v$

$v_{2f} - v_{1f} = v_{2f} - v = 2v$

$v_{2f} = 3v$!

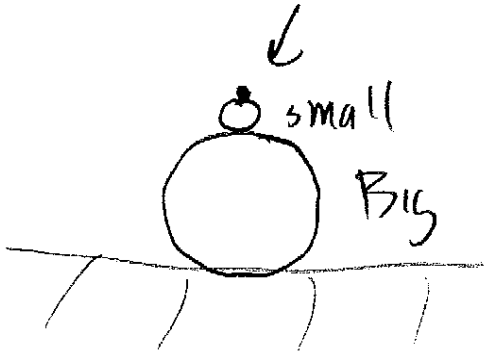


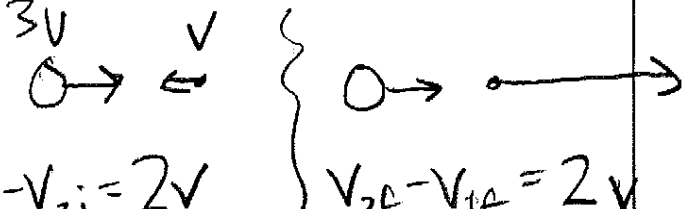
height?

Big ball: $\frac{1}{2}Mv^2 = Mgh$

$h = \frac{v^2}{2g}$

Small ball: $h_{small} = \frac{(3v)^2}{2g} = 9h$!!





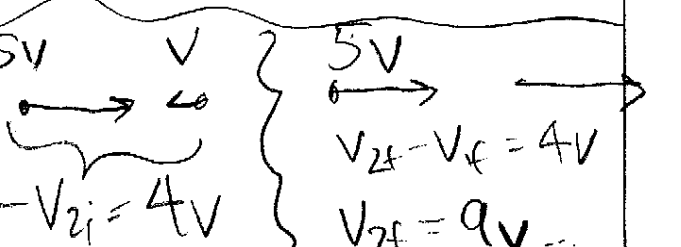
$v_{1i} - v_{2i} = 2v$

$v_{2f} - v_{1f} = 2v$

$v_{2f} = 2v + 3v$

$v_{2f} = 5v$

4



$v_{1i} - v_{2i} = 4v$

$v_{2f} - v_f = 4v$

$v_{2f} = 9v$ ...

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More generally...

want V_{1f}? $V_{2f} = V_{1i} - V_{2i} + V_{1f}$

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 (V_{1i} - V_{2i} + V_{1f})$$

$$(m_1 - m_2) V_{1i} + 2 m_2 V_{2i} = (m_1 + m_2) V_{1f}$$

$$V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

"weighted mean"

"total transfer"

(1) $m_1 = m_2$ ... NO V_{1i} ends up in V_{1f}; all V_{2i} ends up in V_{1f}; $V_{1f} = V_{1i}$

(2) $m_1 >> m_2$

$$V_{2f} = \frac{(m_2 - m_1)}{m_1 + m_2} V_{2i} + \frac{2m_1}{m_1 + m_2} V_{1i}$$

(1 ⇔ 2)

"total transfer"

(1) $m_1 = m_2$, $V_{2f} = V_{1i}$ all V_{1i} in V_{2f}; no V_{2i} in V_{2f}

(2) $m_1 >> m_2$

$$V_{2f} = -V_{2i} + 2V_{1i}$$

see the 3V?

$$-(-V) + 2V = 3V$$

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