Physics 21 Problem Set 9

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due Monday, March 7 at 5pm

Course Announcements:

Reading for these Problems: KK Note 10.1 (pp. 433-437), KK sections 10.1-10.3, RHK4 15-8 and 15-9.

PSR Fellows, who are advanced Physics Majors, are available to help you in the PSR Wed. & Thurs. from 6-8pm, and Sunday in 1640 Broida, 6-8pm.

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1. Simplify these expressions two different ways; first without using polar notation, and second by initially converting the complex number to its polar form $re^{i\theta}$ and taking in from there... in the second way you can leave the final answer in polar form. Then make a plot in the complex plane of the answer and make sure the two ways agree.

   (a) $\dfrac{1}{1+i}$

   (b) $\dfrac{5}{-3+4i}$

   (c) $\dfrac{2}{1-\sqrt{3}i}$

2. Use De Moivre's theorem ($e^{i\theta} = \cos\theta + i\sin\theta$), the half-angle formulas $2\cos^2(\theta/2) = 1 + \cos\theta$ and $2\sin^2(\theta/2) = 1 - \cos\theta$, $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$, and of course $(e^{i\theta})^\star = e^{-i\theta}$ to simplify the following to the general form $x + iy$, meaning, solve for x and y in terms of θ:

   (a) $\frac{1}{2}(e^{i\theta} + e^{-i\theta})$

   (b) $\frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

   (c) $\dfrac{1}{1+e^{i\theta}}$

   (d) $\dfrac{1}{1-e^{i\theta}}$

   (e) In this one, start from $(e^{i\theta})^2 = e^{2i\theta}$ and use De Moivre's theorem on both sides to derive the two-angle formulas for both $\sin 2\theta$ and $\cos 2\theta$.

3. (RHK4 15.62) For the system shown in Fig. 1, the block has a mass of 1.52 kg and the force constant is 8.13 N/m. The frictional force is given by $-b\dot{x}$, where $b = 0.227\,\text{kg/s}$. Suppose that the block is pulled aside a distance 12.5 cm and released.

   (a) Compute $\omega_0 = \sqrt{k/m}$, γ, and ω_1 numerically; this notation is KK. RHK4 denotes by ω' what KK labels ω_0. Is the system overdamped, underdamped, or critically damped?

   (b) Calculate the time interval required for the amplitude to fall to one-third of its initial value.

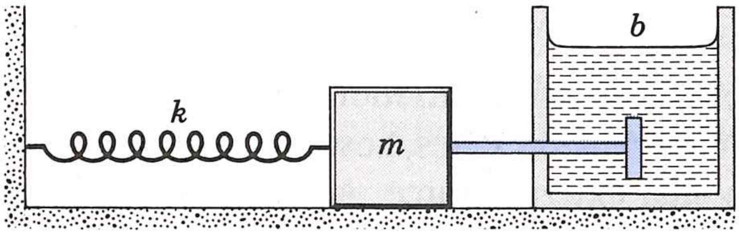

Figure 1: Problem 3.

(c) How many oscillations (full oscillations, not radians) are made by the block in this time?

(d) What b would make the system critically damped?

4. (RHK4 15.64) A damped harmonic oscillator involves a block ($m = 1.91\,\text{kg}$), a spring ($k = 12.6\,\text{N/m}$), and a damping force $F = -b\dot{x}$. Initially, it oscillates with an amplitude of 26.2 cm; because of the damping, the amplitude falls to three-fourths of this initial value after $\nu =$ four complete cycles.

   (a) What is b in terms of the given quantities, and numerically? You may use the approximation that ω_1(KK)$=\omega'$(RHK4)$=\sqrt{k/m} = \omega_0$(KK).

   (b) How much energy is lost during these four cycles?

5. KK 10.3

6. Express the maximum amplitude of a driven oscillator in terms of the maximum force of the the driver F_0, the quality factor Q, and the spring constant k of the driven oscillator.