Physics 21 Problem Set 6

Harry Nelson

due Monday, Feb. 14 at 5pm

Course Announcements:

Reading for these Problems: RHK4 13-1 through 13-5 pp. 271-285, KK 6.2-6.3, pp. 233-247.

PSR Fellows, who are advanced Physics Majors, are available to help you in the PSR Wed. & Thurs. from 6-8pm, and Sunday in 1640 Broida, 6-8pm.

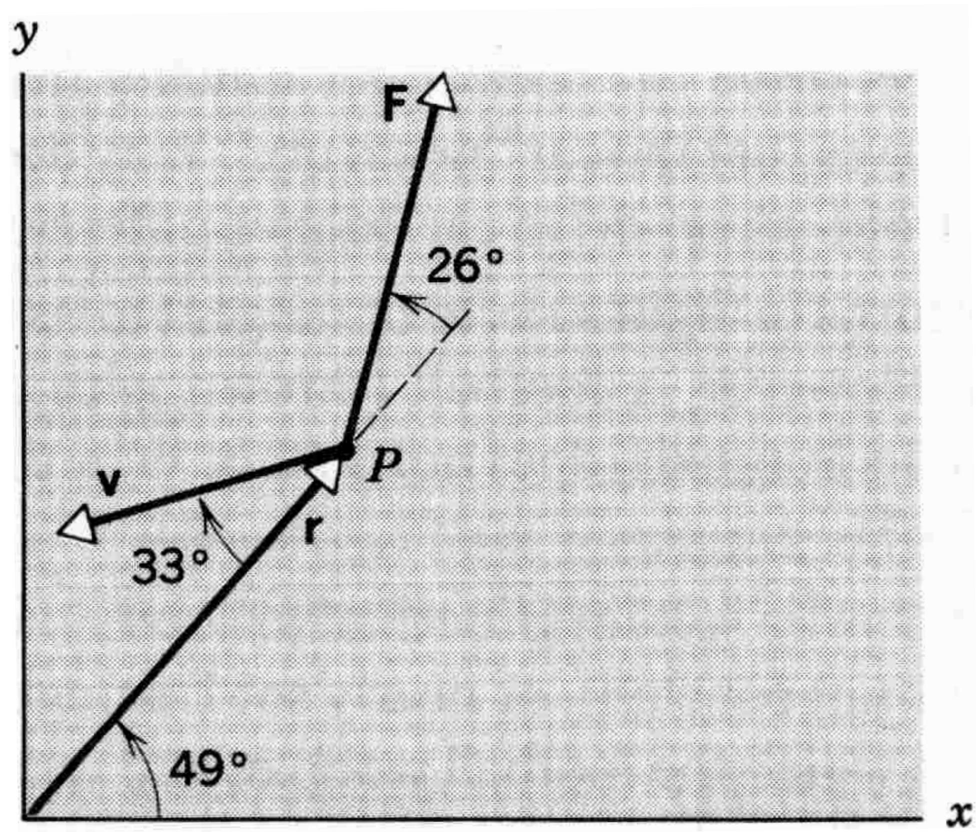

Figure 1: Problem 1.

1. (RHK4 13.2) A particle P with a mass 2.13 kg has position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ as shown in Fig. 1. The particle is acted on by the force $\boldsymbol{F}$. All three vectors lie in a common plane. Presume that $r = |\boldsymbol{r}| = 2.91\,\mathrm{m}$, $v = |\boldsymbol{v}| = 4.18\,\mathrm{m/s}$, and $F = |\boldsymbol{F}| = 1.88\,\mathrm{N}$. Compute the magnitude and directions of:
   (a) the angular momentum of the particle, and
   (b) the torque, about the origin, acting on the particle.

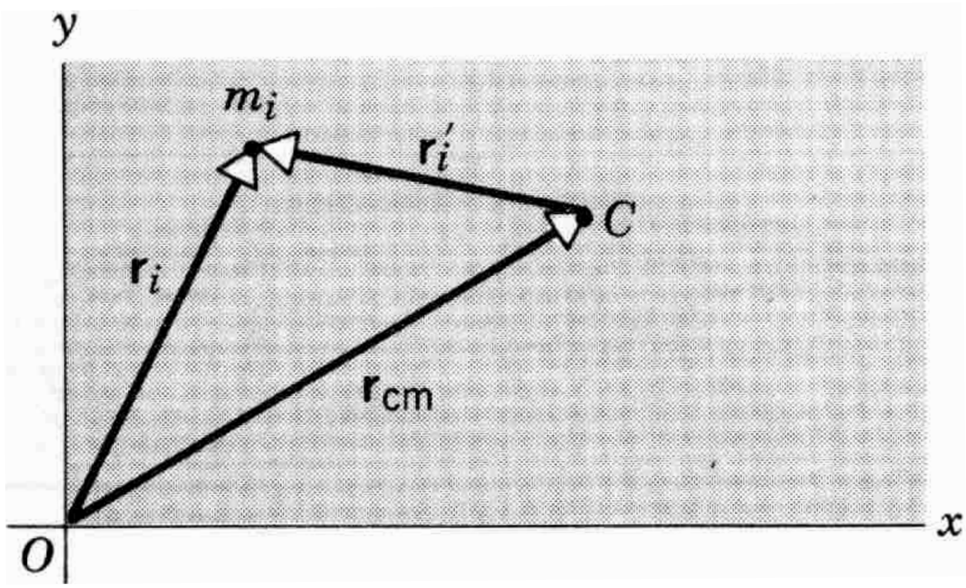

Figure 2: Problem 2.

2. (RHK4 13.7) This is a problem about the distinction between *orbital* angular momentum, which is like that of the Earth in orbit around the Sun, and *spin*, which is like the rotation of the Earth about its axis. Refer to the coordinates shown in Fig. 2; you have to use your imagination to visualize many different small masses m_i, which you should think of being in a rigid body with total mass $M = \sum_i m_i$. The total angular momentum of this system of particles relative to the origin O of an inertial reference frame is $\boldsymbol{J} = \sum_i (\boldsymbol{r}_i \times \boldsymbol{p}_i)$, where the position vector of each mass m_i $\boldsymbol{r}_i$ is measured with respect to O and the momentum vector of each mass m_i is $\boldsymbol{p}_i$. Sometimes in quantum mechanical discussions this quantity is called 'total $\boldsymbol{J}$'.

   (a) Change variables to 'center of mass' quantities $\boldsymbol{r}_i'$ and $\boldsymbol{p}_i'$, which are defined relative to the center of mass C and its motion, where $\boldsymbol{r}_{\text{cm}} = (\sum_i m_i \boldsymbol{r}_i)/M$ and $\boldsymbol{v}_{\text{cm}} = (\sum_i m_i \boldsymbol{v}_i)/M$, and then $\boldsymbol{r}_i = \boldsymbol{r}_{\text{cm}} + \boldsymbol{r}_i'$ and $\boldsymbol{p}_i = m_i \boldsymbol{v}_{\text{cm}} + \boldsymbol{p}_i'$. Prove that $\sum_i m_i \boldsymbol{r}_i' = 0$, and $\sum_i \boldsymbol{p}_i' = 0$.

   (b) Defining the angular momentum of all the particles about their center of mass, which is the spin, as $\boldsymbol{S} = \sum_i (\boldsymbol{r}_i' \times \boldsymbol{p}_i')$, and the orbital angular momentum of the center of mass about O as $\boldsymbol{L} = \boldsymbol{r}_{\text{cm}} \times M\boldsymbol{v}_{\text{cm}}$, starting from $\boldsymbol{J} = \sum_i (\boldsymbol{r}_i \times \boldsymbol{p}_i)$, show after a few lines of algebra that $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$.

   (c) Depict the relationship $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ in a diagram showing the object rotating about C and orbiting about O, with the vectors $\boldsymbol{J}$, $\boldsymbol{L}$, and $\boldsymbol{S}$ shown.

3. (RHK4 13.10) A sanding disk with rotational inertia 1.22×10^{-3} kg m^2 is attached to an electric drill whose motor delivers a torque of 15.8 N m. Find:

   (a) the angular momentum, and

   (b) the angular speed of the disk

   33.0 milliseconds after the disk started from rest.

4. (RHK4 13.9) The time integral of the torque is called the *angular impulse*.

   (a) Starting from $\boldsymbol{\tau} = d\boldsymbol{L}/dt$,show that the resultant angular impulse equals the change in angular momentum. This is the rotational analog of the linear impulse-momentum relation.

(b) For rotation around a fixed axis, show that:

$$\int \tau dt = \langle Fr_\perp \rangle (t_f - t_i) = I(\omega_f - \omega_i),$$

where $\langle Fr_\perp \rangle$ is the average value of the torque during the time interval $\Delta t = t_f - t_i$ when the force F at moment arm $r_\perp$ acts on the body, ω_i and ω_f are the angular velocities of the object (which has rotational inertia I) just prior to and just after application of the force/torque. It is a good idea to think through and write out the equation that defines $\langle Fr_\perp \rangle$.

5. KK 6.4.

6. KK 6.6. Assume that the inner foot of the person is just at radius R from the center of railroad track's circle. The distribution of the net force of friction between the two feet does not matter when computing the torque of friction.

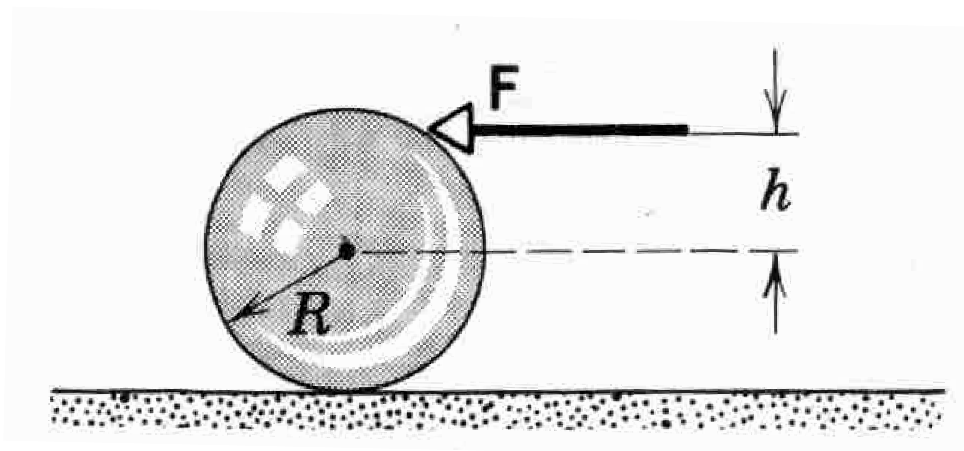

Figure 3: Problem 7.

7. (RHK4 13.23) A billiard ball, initially at rest, is given a sharp impulse by a cue. (*Hint*: ponder whether the linear impulse and rotational impulse are proportional to one another). The cue is held horizontally a distance h above the centerline as shown in Fig. 3. The ball leaves the cue with a speed v_0, and, because of its "forward english," actually speeds up after being struck by the cue, and finally acquires a speed of $9v_0/7$. (*Hint*: ponder whether the initial impulses leave the ball rolling without slipping, and if the ball is spinning and slipping, think what direction the force of kinetic friction acts). Show that $h = 4R/5$, where R is the radius of the ball. (*Hint...* friction does one thing to the translational motion of the ball, and another thing to the rotational motion. After a certain amount of time, the velocity and angular velocity are just right to roll without slipping). This type of strike with the cue was called 'putting follow' on the ball when I used to play pool. Hitting the ball much lower was called 'putting stop' on the ball. A little googling shows me these phrases are still in use.