Physics 21 Problem Set 4

Harry Nelson

due Monday, Jan. 31 at 5pm

Course Announcements: The reading for this problem set is RHK4 Chapter 11 sections 11-4 through 11-6; KK 7.1 and 7.2 pp 288-292; RHK4 Chapter 12 sections 12-1 through 12-3; KK midway p. 249 through 252. The table on page 251 or RHK4 is reproduced on page 4 of this problem set.

The midterm date has been changed to Monday, Feb. 7.

PSR Fellows, who are advanced Physics Majors, are available to help you in the PSR Wed. & Thurs. from 6-8pm, and Sunday in 1640 Broida, 6-8pm.

---

1. In this problem, we look a bit at the definition of the 'Solar Day' for the two inner planets.

   (a) The planet Mercury completes one rotation, relative to the stars far distant from the Solar System, once every 58.646 Earth days; this is called the sidereal period. Mercury completes an orbit around the Sun once every 87.969 Earth days, this is called the orbital period or 'Mercury year'. The axis of Mercury's rotation is just about perfectly perpendicular to the plane of its orbit about the Sun (so Mercury has no seasons), and Mercury's direction of rotation is in the same direction as its orbit about the sun. If you were sitting on the surface of Mercury, how long is a Mercury day, which would be the duration between sunrises, as measured in Earth days? Which is longer, the Mercury year or the Mercury Day? (The MESSENGER mission will commence orbit of the planet Mercury on March 18, 2011... `http://messenger.jhuapl.edu/`).

   (b) Venus has a sidereal period of 243.019 Earth days, and an orbital period of 224.701 Earth days. The axis of rotation of Venus is also just about perfectly perpendicular to the plane of its orbit, however, Venus' direction of rotation is opposite to the direction of its orbit around the Sun. What is the duration of a Venus day as measured in Earth days?

2. A point on a rotating body moves in the xy plane such that $x = R\cos\omega t$ and $y = R\sin\omega t$. Here x and y are the coordinates of the point, t is the time, and R and ω are constants. (RHK4 11.41)

   (a) Eliminate t between these equations to find y as a function of x, which is the equation of the curve in which the point moves. What is this curve?

   (b) Describe the direction and magnitude of the vector $\vec{\omega}$. Evaluate $\vec{\alpha} = d\vec{\omega}/dt$.

   (c) Find the velocity vector $\vec{v}(t)$ of the point two different ways, and verify you get the same answer either way:

      i. By differentiating $\vec{r}(t) = x(t)\hat{\boldsymbol{\imath}} + y(t)\hat{\boldsymbol{\jmath}}$.

      ii. By computing $\vec{\omega} \times \vec{r}(t)$. Recall that $\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\jmath}} = -\hat{\boldsymbol{\imath}}$.

   (d) Evaluate $|\vec{v}(t)|$.

   (e) Find the acceleration vector $\vec{a}(t)$ of the point two different ways, and verify you get the same answer either way:

i. By differentiating $\vec{v}(t)$ found in the last part.

ii. By computing $\vec{\alpha} \times \vec{r}(t) + \vec{\omega} \times \vec{v}(t)$.

(f) Evaluate $|\vec{a}(t)|$.

3. Calculate the rotational inertia of a meter stick, with mass 0.56 kg, about an axis perpendicular to the stick and located at the 20-cm mark. Use the information in Figure 9 on page 251 of RHK4. (RHK4 12.7)

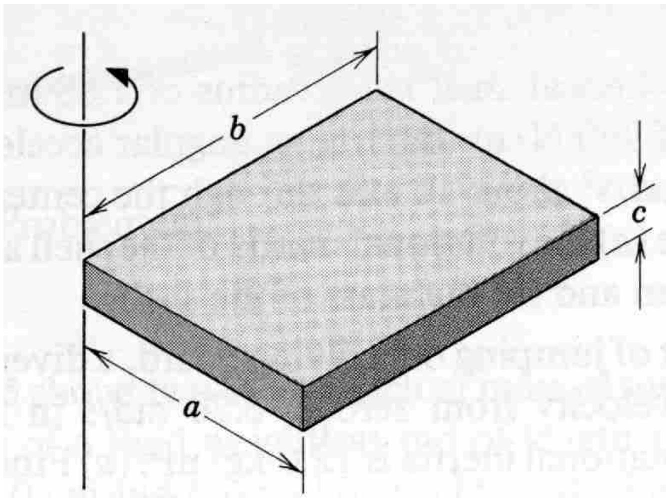


Figure 1: Problem 4.

4. Fig. 1 shows a uniform block of mass M and edge lengths a, b, and c. Calculate its rotational inertia about an axis through one corner and perpendicular to the large face of the block. Use the information in Figure 9 on page 251 of RHK4. (RHK4 12.6)

5. This is RHK4 12.11:

   (a) Show that a solid cylinder of mass M and radius R is equivalent to a thin hoop of mass M and radius $R/\sqrt{2}$, for rotation about the axis of symmetry.

   (b) More generally, the radial distance from a given axis at which the mass of a body could be concentrated without altering the rotational inertia of the body about that axis is called the *radius of gyration*. Let k represent the radius of gyration and show that $k = \sqrt{I/M}$. This gives the radius of the "equivalent hoop" in the general case.

6. In this problem we seek to compute the rotational inertia of a disk of mass M and radius R about an axis through its center and perpendicular to its surface. Consider a mass dm in the shape of a ring of radius r and width dr (see Fig. 2).

   (a) What is the mass dm of this element, expressed as a fraction of the total mass M of the disk?

   (b) What is the rotational inertia dI of this element?

   (c) Integrate the result of the previous part to find the rotational inertia of the entire disk.

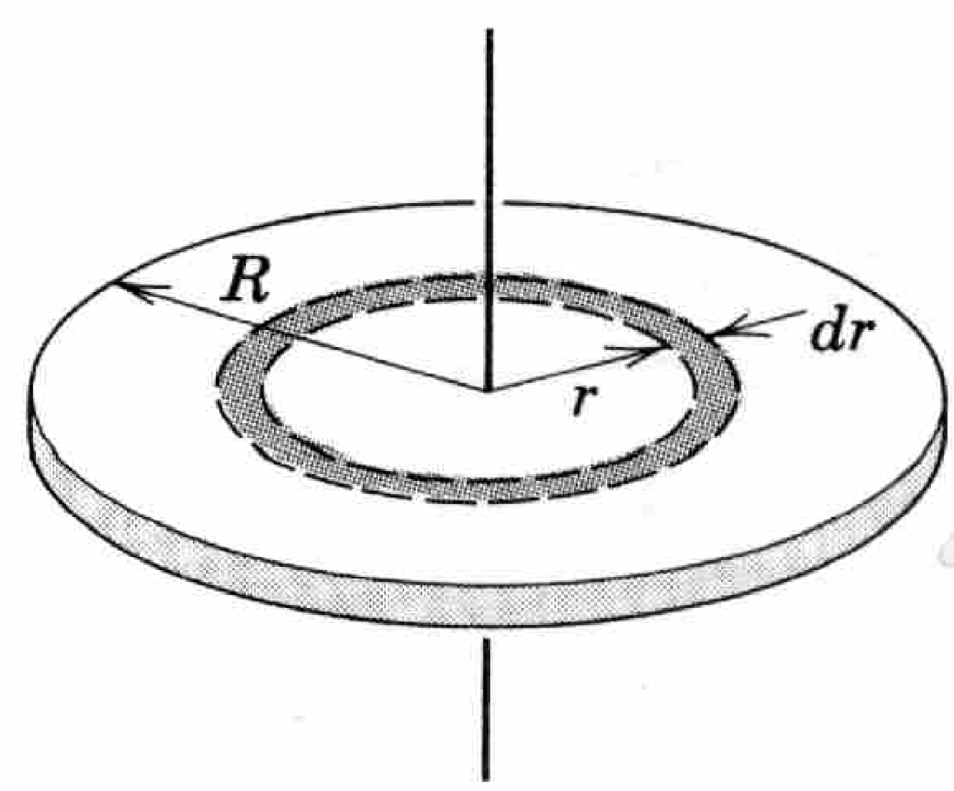


Figure 2: Problem 6.

7. Five solids are shown in are shown in cross section in Fig. 3. The cross sections have equal heights and equal maximum widths - this means that the *diagonal* of the face of the cube and the *altitude* of the face of the equilateral triangular prism equal the diameter of the round objects. The solids have equal masses. Rank the objects by rotational inertia about the axis shown by the dot. You should use the information in Figure 9 on page 251 or RHK4.. however, the triangular prism is not in the table, and is hard. Make your best argument for it... there is a clever way to get the triangle's rotational inertia without doing any integrals, but it is devious... (RHK4 12Q6).

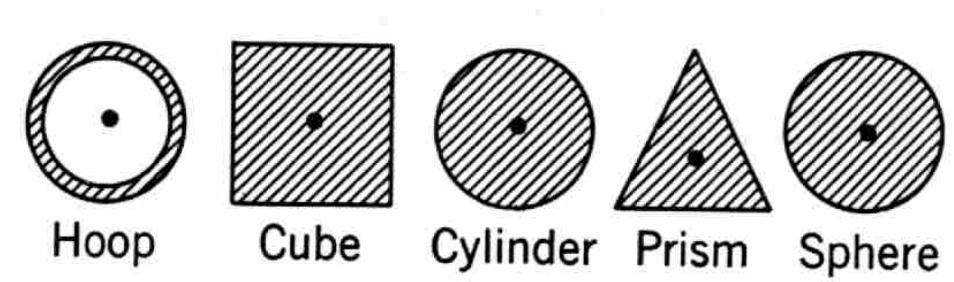
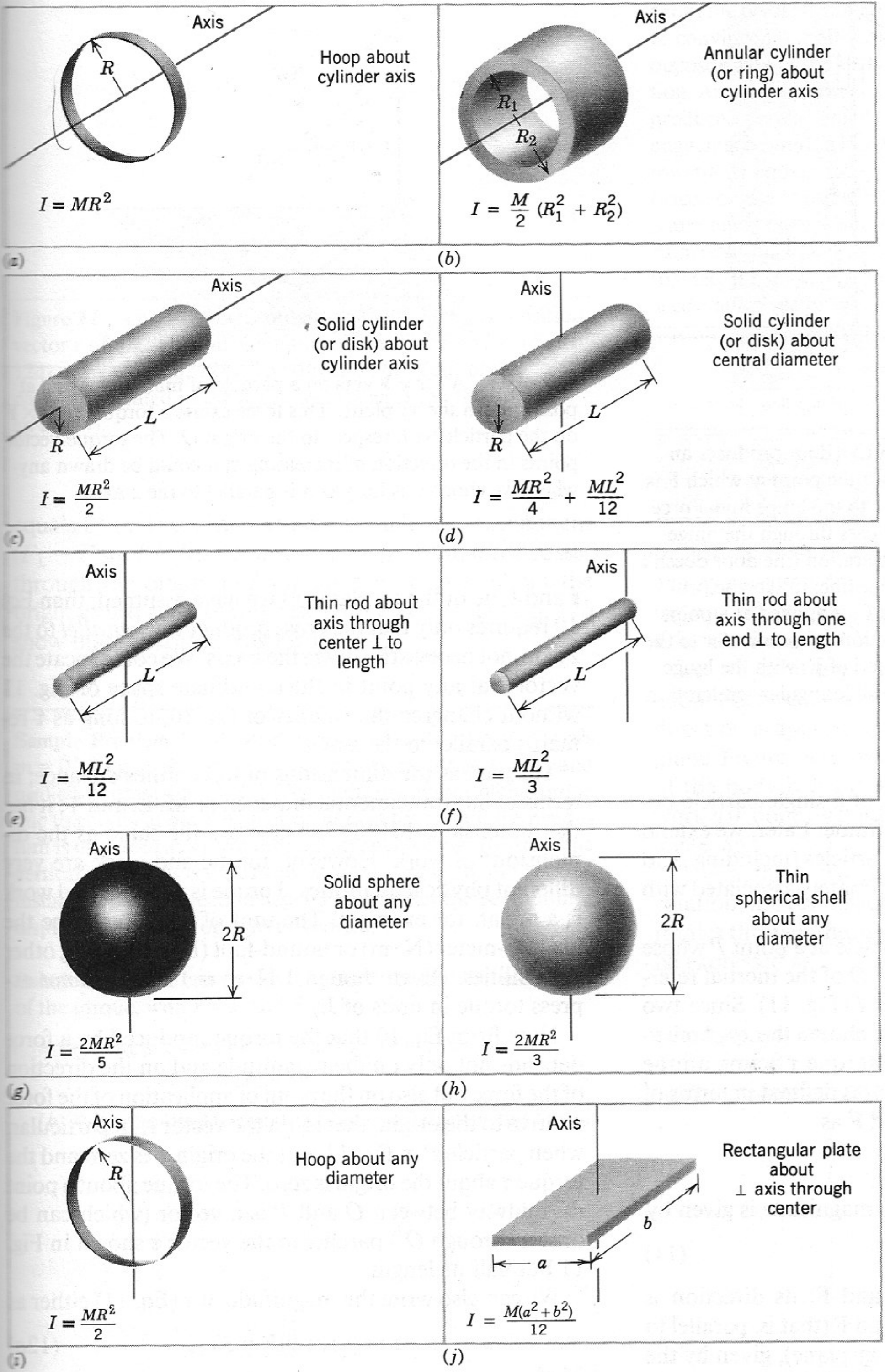


Figure 3: Problem 7.



Axis
R
Hoop about cylinder axis
$I = MR^2$
(a)
Axis
R_1
R_2
Annular cylinder (or ring) about cylinder axis
$I = \frac{M}{2}(R_1^2 + R_2^2)$
(b)
Axis
L
R
Solid cylinder (or disk) about cylinder axis
$I = \frac{MR^2}{2}$
(c)
Axis
L
R
Solid cylinder (or disk) about central diameter
$I = \frac{MR^2}{4} + \frac{ML^2}{12}$
(d)
Axis
L
Thin rod about axis through center ⊥ to length
$I = \frac{ML^2}{12}$
(e)
Axis
L
Thin rod about axis through one end ⊥ to length
$I = \frac{ML^2}{3}$
(f)
Axis
2R
Solid sphere about any diameter
$I = \frac{2MR^2}{5}$
(g)
Axis
2R
Thin spherical shell about any diameter
$I = \frac{2MR^2}{3}$
(h)
Axis
R
Hoop about any diameter
$I = \frac{MR^2}{2}$
(i)
Axis
b
a
Rectangular plate about ⊥ axis through center
$I = \frac{M(a^2+b^2)}{12}$
(j)

Fi
va