

# Physics 21 Problem Set 3

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due Monday, Jan. 24 at 5pm

**Course Announcements:** The reading for this problem set is KK Chap. 4, pp. 187-194, and RHK4 Chapter 10; the material is mostly collisions, with a little on escape velocity. We will follow both texts this week in lecture.

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1. In this problem work out the mass and radius of an asteroid with the same density as that of the Earth, but, that is small enough that you could jump off of it and attain escape velocity.

- (a) Show that the mass  $m$  of a spherical asteroid, radius  $r$ , with the same mass density as that of the Earth can be written:

$$m = M_E \times \left( \frac{r}{R_E} \right)^3$$

where  $M_E = 6 \times 10^{24}$  kg is the mass of the Earth, and  $R_E = 6.4 \times 10^6$  m is the radius of the Earth.

- (b) Suppose you can jump from the Earth's surface up a distance  $h = 1$  m. Show that the radius of a spherical asteroid with the same mass density of the Earth from which you could jump and achieve escape velocity is  $r = \sqrt{hR_E}$ , and numerically evaluate  $r$  and  $m$ , the mass of the asteroid. Is the mass of the asteroid comparable to a typical human mass of 50 kg, much bigger, or much smaller?
  - (c) Just in case, imagine that point masses  $m_1$  and  $m_2$  are initially at rest a distance  $r$  apart. The masses push off from one another with entirely internal forces, so that total momentum (initially zero) is conserved. What must be the initial velocity of  $m_1$  in order that the two masses eventually separate by an infinite distance? That is, what is  $m_1$ 's escape velocity? You can not assume the limit  $m_1 \ll m_2$ .
2. In Fig. 1, the pendulum bob has mass  $m_1 = 0.5$  kg, and undergoes an elastic collision with mass  $m_2 = 1$  kg on the table. The length of the pendulum is  $L = 0.5$  m. Find the final velocity of the mass on the table, and the maximum angle from the downward vertical that the bob makes after the collision. (RHK4 10.32)
  3. One of the bounciest balls in history was the Wham-O Super Ball. They were popular in 1965-1966, and came in three sizes. Assume you took the biggest (the 'normal' Super Ball with radius  $R_1 = 115/16$ " ) and the smallest (the 'Super-Mini-Ball' with radius  $R_2 = 0.78$ " ), put them in contact with the smaller one atop the bigger one, and let them drop on to a perfect floor from a height of  $h = 1$  m. The situation is like the 'basketball amplifier' demonstration done in class. Assume that the density of the special rubber used in the Super Balls, known as 'vectron' is the same for normal and mini-balls. For the distances falling and rising, make the the approximation that the ball's sizes are negligible.

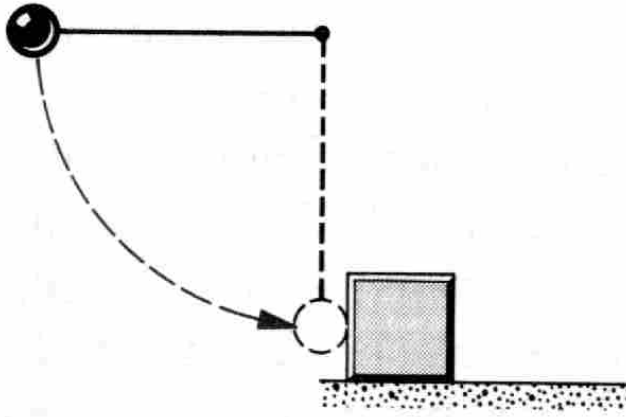


Figure 1: Problem 2.

- (a) To what height will the Super-Mini-Ball rise? Assume that the collisions are perfectly elastic. Symbolically, put your answer in terms of the ratio of  $m_2$  to  $m_1$ ,  $\rho = m_2/m_1$ , and the original height  $h$ , and then calculate the numerical answers.
- (b) In real life, bouncing objects lose a bit of energy when they collide. As you know from class (and from p. 212, Equation (14) of RHK4), for a perfectly elastic collision (symbols as in RHK4):

$$\frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}} = 1.$$

However, for a real collision, the same ratio is less than one, and its value is called the ‘coefficient of restitution’  $c$ . Show that the appropriate relationships for coefficients of restitution not equal to 1 are:

$$v_{1f} = \left( \frac{m_1 - cm_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{(1+c)m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left( \frac{(1+c)m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2 - cm_1}{m_1 + m_2} \right) v_{2i}$$

- (c) For all Super-Balls, the coefficient of restitution is  $c = 0.95$  (ping-pong balls also have a large  $c$ , but are slowed down by air resistance). Now for the ball drop considered earlier, how high does the Super-Mini-Ball rise? Don’t forget that there are two collision; for the first collision, between the Normal ball and the floor, treat the Earth as infinite in mass.

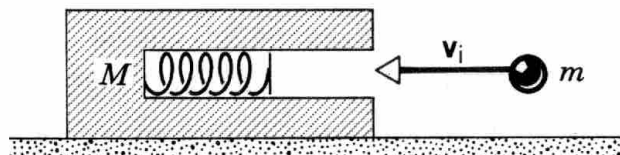


Figure 2: Problem 4.

4. In Fig. 2, the assembly with the spring has mass  $M$  and is initially at rest. The ball (of mass  $m$ ) enters the cavity and compresses the spring until the ball is not moving with respect to the

mass  $M$ ... that is, both  $m$  and  $M$  are, at this instant, are moving with the same velocity. At this instant, what fraction of mass  $m$ 's initial energy is converted to potential energy of the spring? As  $M/m \rightarrow \infty$ , what is the limiting value of this fraction? (RHK4 10.40)

5. A ball of mass  $m_1 = 2$  kg is moving with velocity  $v_{1i} = 3$  m/s toward a stationary ball of mass  $m_2 = 1$  kg. They collide, and  $m_1$  is deflected by the maximum angle possible,  $|\phi_{1m}|$ . Find, numerically (with the help of pictures of the pertinent 'velocity circles'):
- (a)  $|\phi_{1m}|$
  - (b) The angle of  $m_2$ 's recoil from  $m_1$ 's initial direction,  $\phi_2$ .
  - (c) The final speed of  $m_1$ ,  $v_{1f}$ .
  - (d) The final speed of  $m_2$ ,  $v_{2f}$ .
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