Physics 21 Problem Set 10

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due Tuesday, March 15 at 8am (at Final)

Course Announcements:

Reading for these Problems: KK Note 10.1 (pp. 433-437), KK sections 10.1-10.3, RHK4 15-8 and 15-9.

PSR Fellows, who are advanced Physics Majors, are available to help you in the PSR Wed. & Thurs. from 6-8pm, and Sunday in 1640 Broida, 6-8pm.

The Final is Tuesday, March 15 at 8:00am in 1640 Broida!

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1. Let's analyze carefully the forced, damped oscillator we observed in class. The mass was $m = 0.25\,\mathrm{kg}$, and the period $T = 1/2\,\mathrm{s}$. It took 100 periods for the amplitude to decay to $e^{-\pi} = 0.043$ of its initial value.

   (a) Compute ω_1, numerically... since damping was present, you are calculating ω_1, not ω_0, from T. Refresh yourself on notation from KK if you must.

   (b) Find Q, and then γ, assuming that $\omega_0 = \omega_1$.

   (c) Actually, ω_0 is a little bigger than ω_1. Physically, why is ω_0 bigger?

   (d) Make the *assumption* that γ from the previous portion is sufficiently accurate, and use the expressions from KK to numerically compute ω_0, and also compute the *fractional* change $\delta = (\omega_0 - \omega_1)/\omega_1$. How does δ compare to $1/Q$? You should find that one of them is very much smaller than the other, which justifies the approximation that $\omega_0 = \omega_1$ used above.

   (e) Now assume there is a driving force $F_0 \cos \omega t$. Assume that $F_0 = 0.25\,\mathrm{N}$.

      i. If the mass on the spring got pulled to a new static equilibrium by F_0, how far would it be displaced from its previous equilibrium position, in millimeters?

      ii. If the driving force is modulated at the (circular) frequency $\omega = \omega_0$, what is the amplitude of the driven oscillation in meters?

      iii. Now consider the general response of the driven oscillator to F_0 modulated at the (circular) frequency ω. Drive the following expression for the displacement from equilibrium:

      $$x(t) = \xi(\omega) \cos(\omega t - \phi(\omega)).$$

      Here $\xi(\omega)$ and $\phi(\omega)$ are functions of ω that involve the parameters ω_0 and γ, which you need to do a little bit of complex-valued arithmetic to determine. Make very careful numerical plots of $\xi(\omega)$ and $\phi(\omega)$, for ω from 0 (static) to $10\omega_0$. Be really careful and do a great job when $\omega \approx \omega_0$!! You will need to use lots of closely spaced points there. You don't have to make many closely spaced points far from $\omega \approx \omega_0$. Graphically calculate the Full Width at Half Maximum. By the way, at least one great discovery (which eventually won a Nobel Prize) was missed because the experimenters didn't plot enough points near the resonance.
