$$P_{Hand,\,final} - P_{Hand,\,initial} = \frac{m v_0^2}{l}\,, T$$

$$\boxed{F = \frac{dP}{dT} = \frac{m v_0^2}{l}}$$

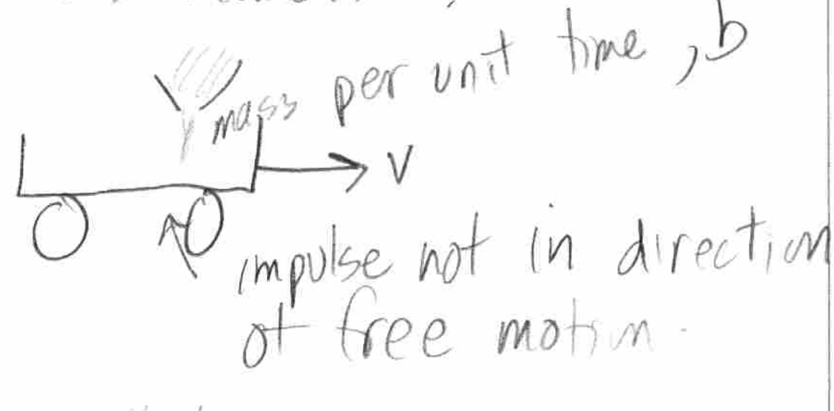
$F \propto v^2$
important.

<u>Hydrant</u>: Compute how much water leaves, now much momentum ($\propto T$), differentiate.

does $\vec{P}_{final} - \vec{P}_{initial} = \int \vec{F}\,dt$ ever fail?

sometimes.. when new mass carries an impulse that influences momentum.

Formula OK?




$\frac{d}{dt}\big((M+m-bt)v\big) = F$


$(M+m-bt)v = Ft + \text{constant}$ b mass unit time.

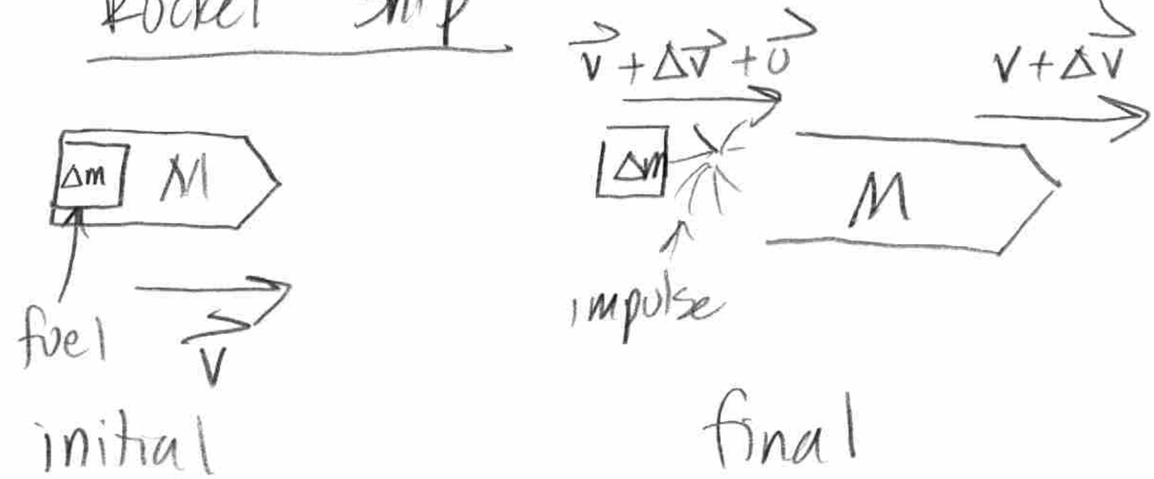
$t = 0, \quad v = v_0$

$(M+m)\cdot v_0 = \text{constant}$

$$\boxed{v = \frac{Ft + (M+m)\cdot v_0}{M+m-bt}}$$

Formula Not OK

<u>Rocket Ship</u>




$$\vec{P}_{initial} = (M+\Delta m)\vec{v}$$

$$\vec{P}_{final} = M(\vec{v}+\Delta\vec{v}) + \Delta m(\vec{v}+\underbrace{\Delta\vec{v}}_{\text{ignore}}+\vec{u})$$

$$\Delta\vec{P} = \vec{P}_{final} - \vec{P}_{initial} = M\Delta\vec{v} + \Delta m\vec{u}$$

SYSTEM $\frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{u} = M\frac{d\vec{v}}{dt} - \frac{dM}{dt}\vec{u}$

$\frac{dm}{dt} = -\frac{dM}{dt}$ fuel lost from rocket

Most interesting: Launch a rocket!


$\vec{u} = -u\hat{j} \quad \vec{F}_{ext} = -Mg\hat{j}$

$\vec{v} = v\hat{j}$

$$-Mg = M\frac{dv}{dt} + \frac{dM}{dt}u$$

$$dv = -u\frac{dM}{M} - g\,dt$$

$$v = -u\ln\left(\frac{M}{M_0}\right) - gt$$

$$\boxed{v = u\ln\left(\frac{M_0}{M(t)}\right) - gt}$$

get $M(t)$ small as fast as possible