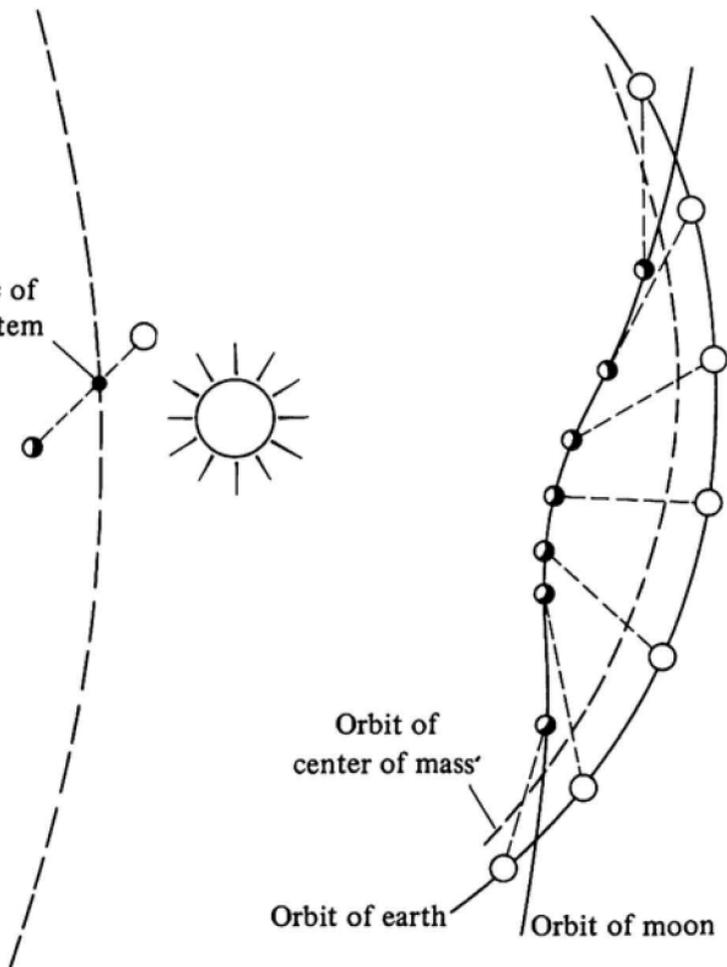


Center of mass of  
earth-moon-sun  
system

Center of mass of  
earth-moon system



Orbit of  
center of mass'

Orbit of earth

Orbit of moon





Moon

Eris

Ceres

Charon

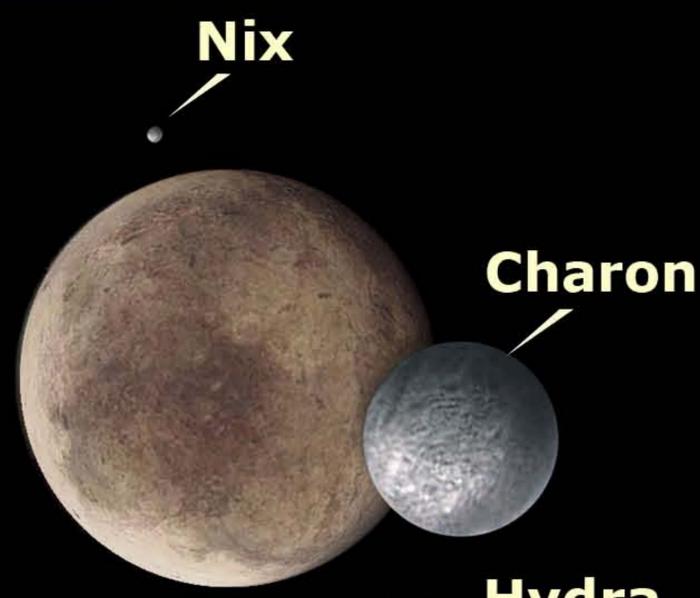
Pluto

# Largest known trans-Neptunian objects (TNOs)



Dysnomia

Eris

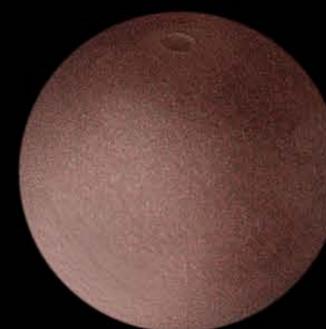


Nix

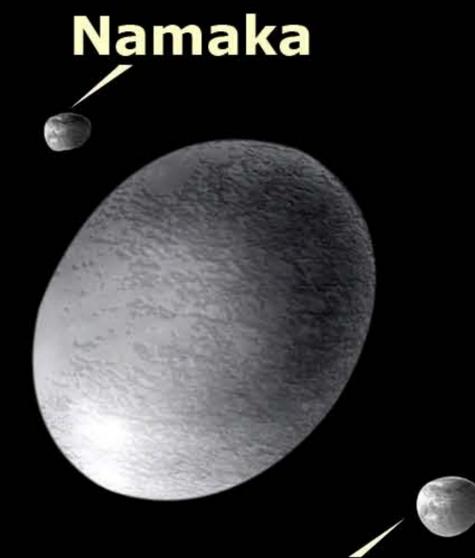
Charon

Hydra

Pluto



Makemake



Namaka

Hi'iaka

Haumea



Sedna



Vanth

Orcus

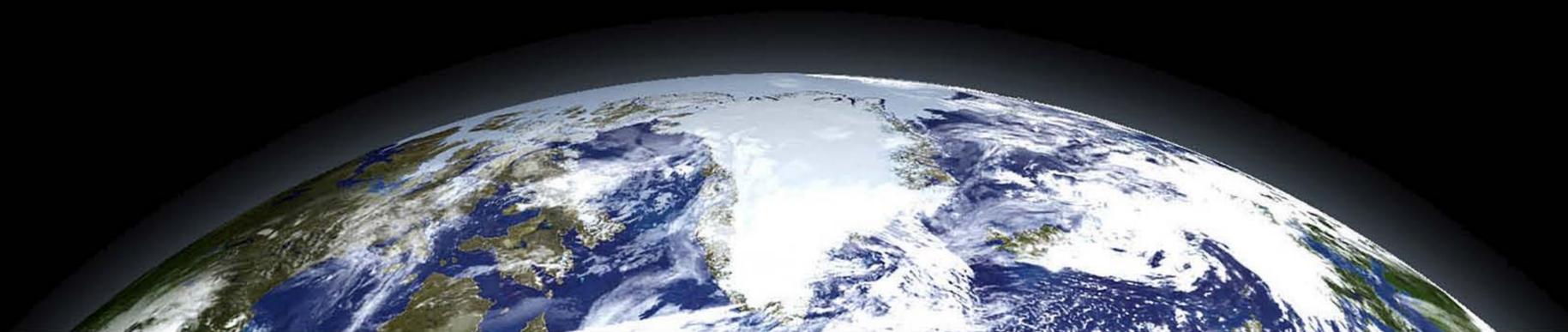


2007 OR<sub>10</sub>



Weywot

Quaoar



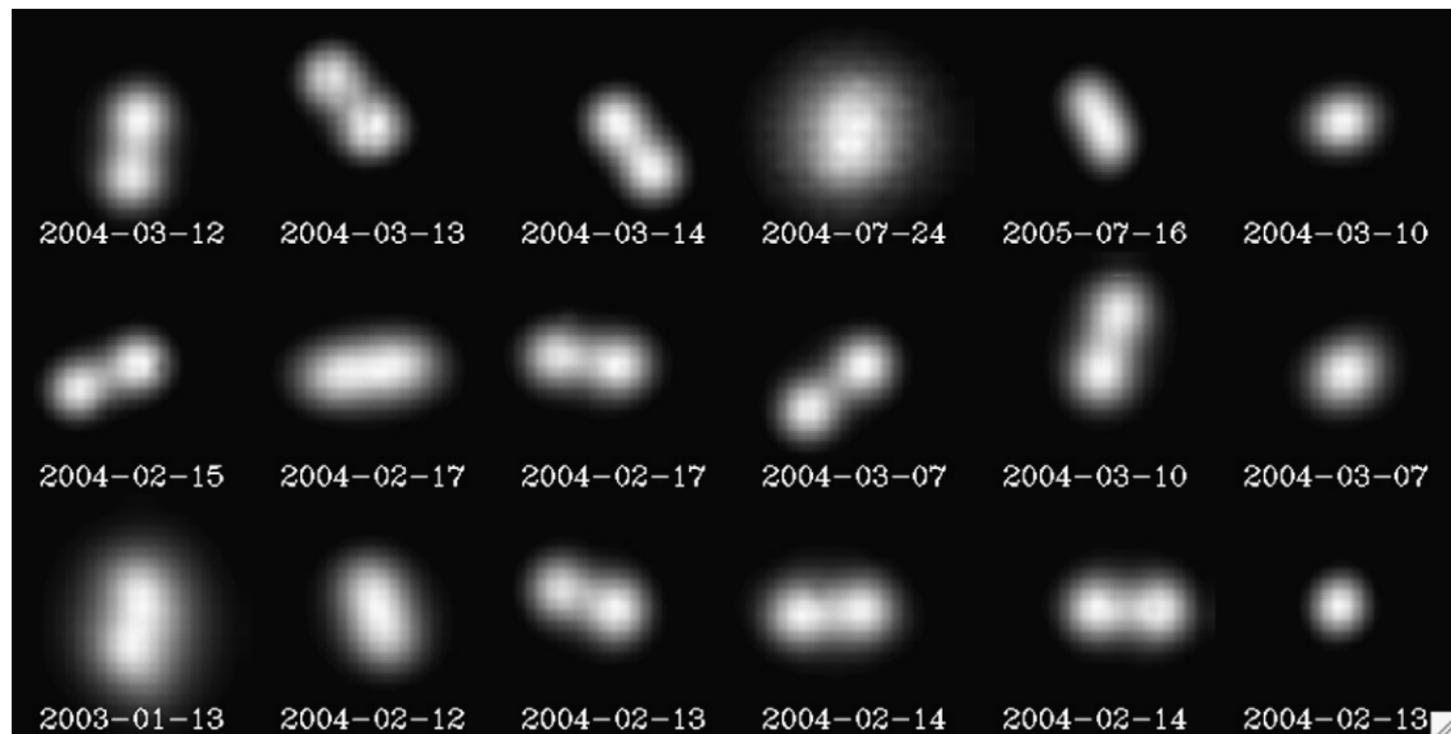
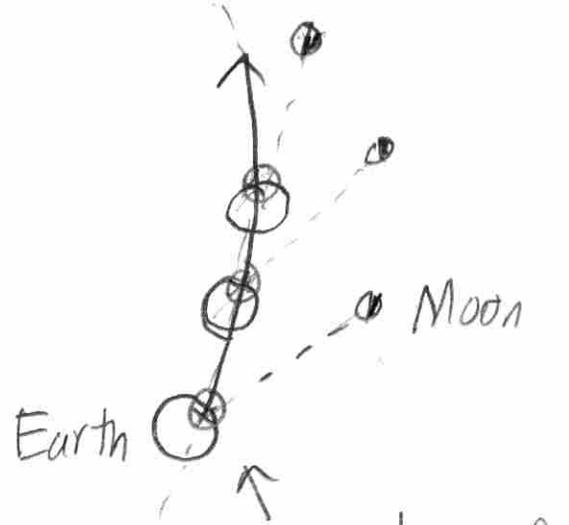


Fig. 1. (90) Antiope system observed with VLT NACO in 2004. The two components of the doublet system are clearly identified on these basic-processed (sky subtraction, flat-fielding, bad pixel removal) near infrared observations. The relative positions of the two components can be found in [Table 1](#). We also displayed on the far right three PSF frames. Because the FWHM of the PSF is similar to the FWHM on the individual components of the double system, we can deduce that the two components cannot be resolved individually by the AO system. Their angular size is below the diffraction limit of the telescope (60 milli-arcsec for the VLT). The July 2004 observation was taken under very poor seeing conditions. In this case, the binary nature of (90) Antiope cannot be revealed.

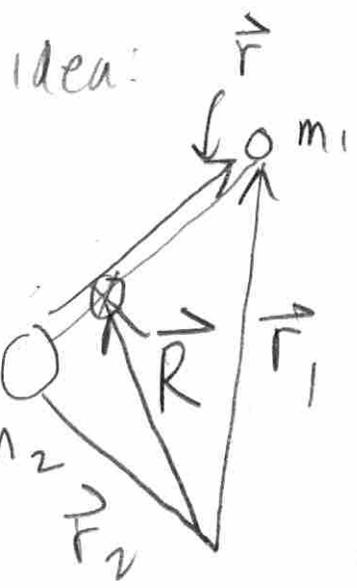


$$M \ddot{\mathbf{R}} = \mathbf{F}_{\text{ext, net}}$$

Center of mass of earth/moon system follows orbit around sun.

Earth-Moon System Rotates about its center of mass... GIF

BINARY ASTEROIDS: (recent)



$$\mu = \frac{m_1 m_2}{m_1 + m_2}, \quad \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2, \quad \hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{r}$$

$$\mu \ddot{\mathbf{r}} = \mathbf{F}_{\text{int}, 1}$$

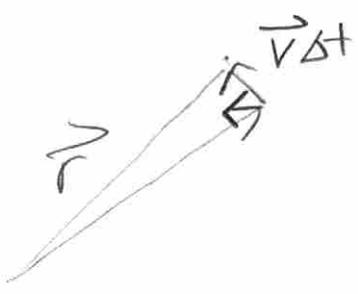
gravity:  $\mathbf{F}_{\text{int}, 1} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}$

$$\frac{m_1 m_2}{m_1 + m_2} \frac{\ddot{\vec{r}}}{r} = -G \frac{m_1 m_2}{r^2} \hat{r}$$

$$\ddot{\vec{r}} = -G \frac{(m_1 + m_2)}{r^2} \hat{r}$$

ASSUME  
CIRCULAR

$$-\frac{v^2}{r} = -\frac{G(m_1 + m_2)}{r^2}$$



$$v = \left[ \frac{G(m_1 + m_2)}{r} \right]^{1/2}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\left[ \frac{G(m_1 + m_2)}{r} \right]^{1/2}}$$

$$T^2 = \frac{(2\pi)^2 r^3}{G(m_1 + m_2)}$$

$$(m_1 + m_2) = \frac{(2\pi)^2 r^3}{G T^2}$$

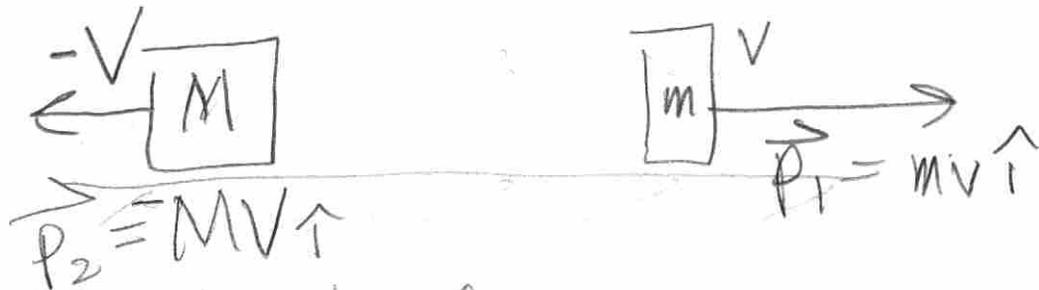
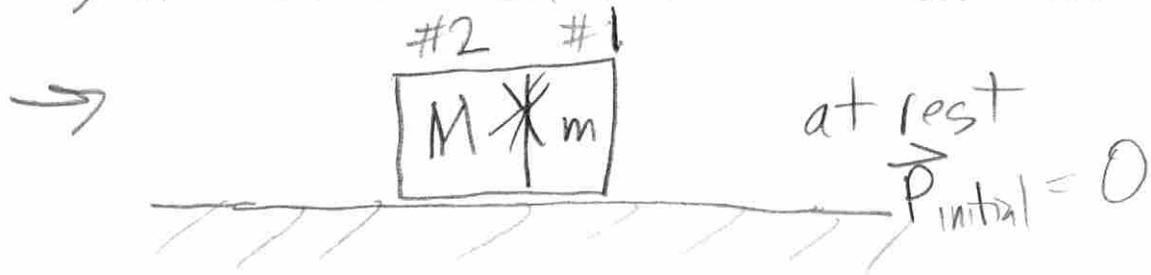
90 Antiope : show figure

$$r = 171 \text{ km} \quad T = 16.5 \text{ hours}$$

$$(m_1 + m_2) = \frac{(2\pi)^2 \cdot (171 \cdot 10^3)^3 \text{ m}^3}{\frac{2}{3} \cdot 10^{-10} \frac{\text{kg} \cdot \text{m}^3}{\text{s}^2 \text{ kg}^2} \cdot (16.5 \cdot 3600)^2}$$

$$(m_1 + m_2) \approx 8.4 \cdot 10^{17} \text{ kg} \quad \text{density} \approx 1.25 \frac{\text{gm}}{\text{cm}^3}$$

→ no CM continuous distribution.



→ only internal forces

$$\frac{d}{dt} (\vec{p}_1 + \vec{p}_2) = 0$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \vec{0}$$

$$-MV + mv = 0$$

$$\frac{v}{V} = \frac{M}{m}$$

Use in  
your  
problem

what did happen. an impulse

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2$$

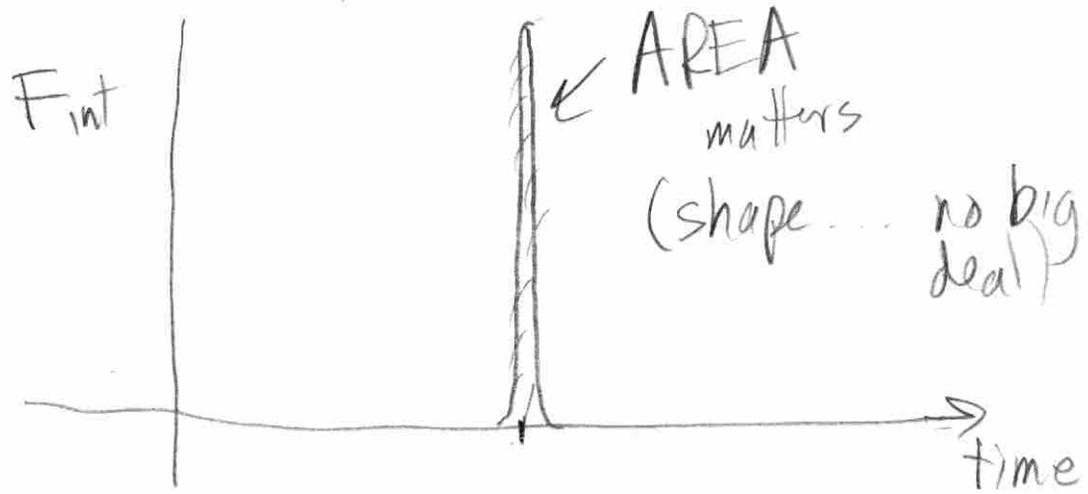
$$N \vec{r}^{\circ\circ} = \vec{F}_{\text{int}}$$

$$\vec{r}(0)=0 \quad \mu(\dot{\vec{r}}(t) - \dot{\vec{r}}(0)) = \int_0^t \vec{F}_{int} dt = \vec{P}_f - \vec{P}_i$$

the impulse  $\vec{I}$  (my symbol)

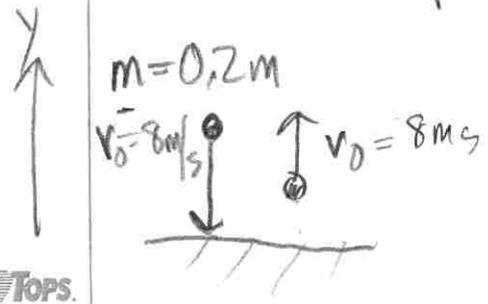
Two types here:

- ①  $\vec{F}_{int}$  from an explosion... large, brief.



OFTEN: ball, person + Earth.  
 $\mu \rightarrow m$        $M_e \rightarrow \infty$

then  $\vec{P}_f - \vec{P}_i = \vec{I}$



$$\vec{P}_i = -mV_0 \hat{j}$$

$$\vec{P}_f = mV_0 \hat{j}$$

Impulse then  $m v_0 \hat{j} - (-m v_0 \hat{j}) = \vec{I}$   
 $= -2 m v_0 \hat{j}$   
 $= -2 \cdot 0.2 \cdot 8 \hat{j} = 3.2 \hat{j} \frac{\text{kg} \cdot \text{m}}{\text{s}}$

$\Delta t = 10^{-3} \text{ s}$

$\vec{F}_{\text{avg}} \cdot \Delta t = 3.2 \hat{j} \frac{\text{kg} \cdot \text{m}}{\text{s}}$

$\vec{F}_{\text{avg}} = \frac{3.2}{10^{-3}} \hat{j} \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$

$\vec{F}_{\text{avg}} = 3200 \hat{j} \text{ N}$  !

Balls' acceleration ...  $\frac{\vec{F}_{\text{avg}}}{m} \approx 16,000 \text{ m/s}^2$

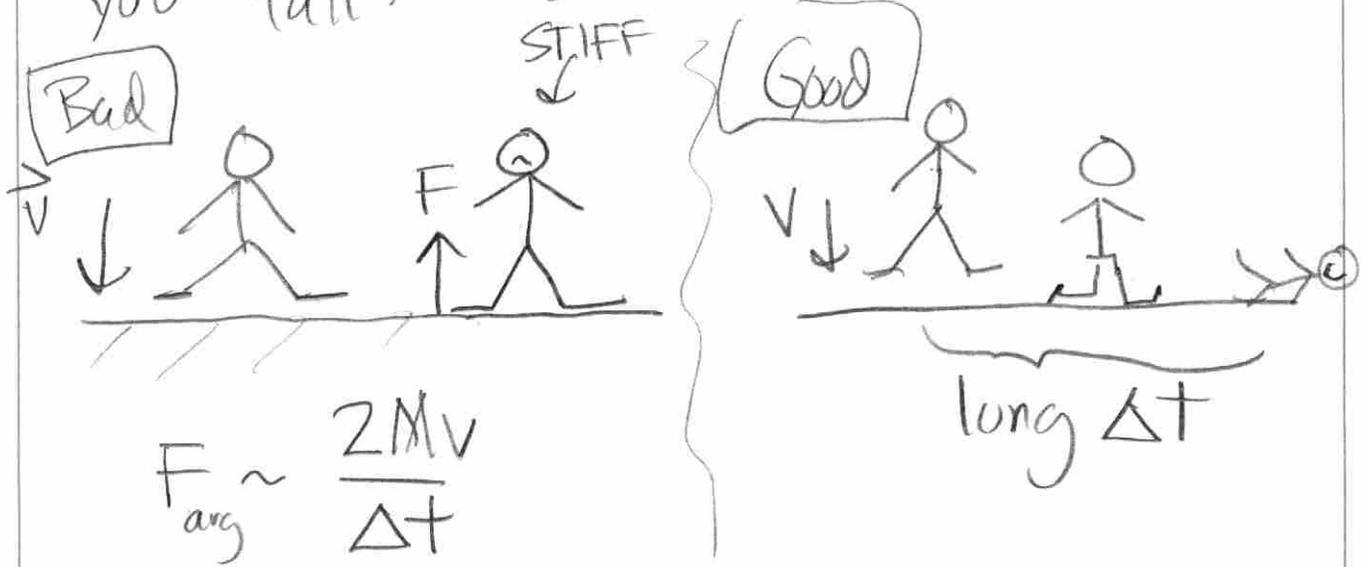
Neglected gravity,

(2) Truly constant force

$F_{\text{grav}} \cdot \Delta t = 0.2 \cdot 9.8 \cdot 10^{-3} \text{ N}$   
 $\approx 2 \cdot 10^{-3} \text{ N}$

Negligible.

To avoid broken limbs, make  $\Delta t$  as long as possible when you fall:



Continuous Tiny Impulses = Force

water



each

loses  $mv_0$

in time  $T$ ,  $\frac{v_0 \cdot T}{l}$  drops hit.

total momentum transfer:

$$= \left( \frac{v_0 T}{l} \right) \cdot (mv_0)$$