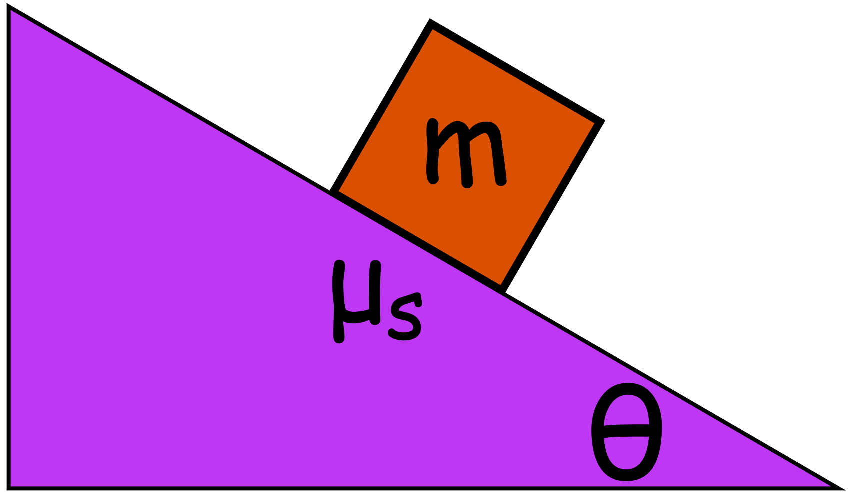


Incline of infinite mass, non-zero static friction. Call the magnitude of the frictional force f and assume the block is not moving. Newton's law in *vertical* is:

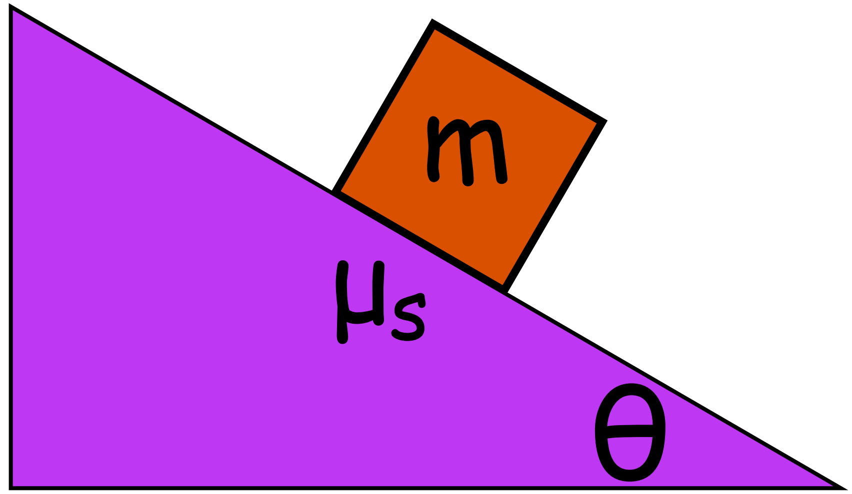
A. $-mg\cos\theta + N = 0$

B. $-mg + N\cos\theta = 0$

C. $N\sin\theta - f\cos\theta = mg\sin\theta$

D. $N\sin\theta = mg\sin\theta$

E. $N\cos\theta + f\sin\theta - mg = 0$



Incline of infinite mass, non-zero static friction. Call the magnitude of the frictional force f and assume the block is not moving. Newton's law in *horizontal* is:

A. $-f + mg\sin\theta = 0$

B. $-f\sin\theta + mg = 0$

C. $N\sin\theta - f\cos\theta = 0$

D. $N\sin\theta - f\sin\theta = mg\cos\theta$

E. $N\cos\theta - mg + f\sin\theta = 0$

$$f_s \leq \mu_s N$$

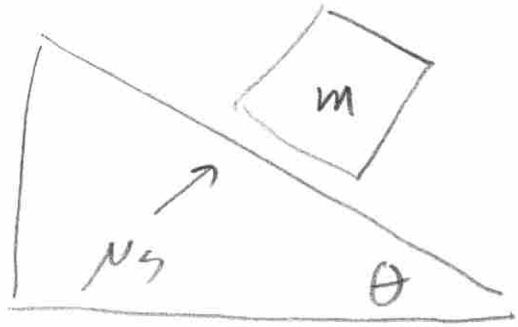
When F exceeds $\mu_s N$, sliding starts..

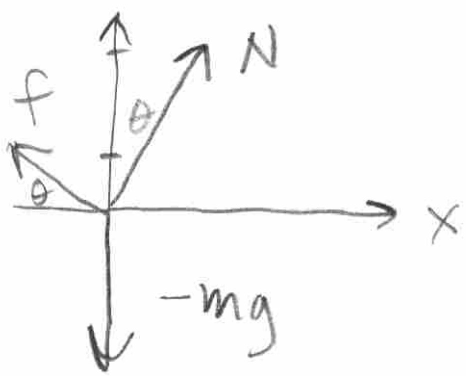
$$f_k = \mu_k N$$

Generally, $\mu_k < \mu_s$

Don't skid !

	μ_s	μ_k
Wood / Wood	0.25-0.5	0.2
Glass / Glass	0.9-1.0	0.4
Steel / Steel clean	0.6	0.6
" lub.	0.09	0.05
Rubber on dry	1.0	0.8
Ski on snow	0.04	0.04
Teflon / Teflon	0.04	0.04





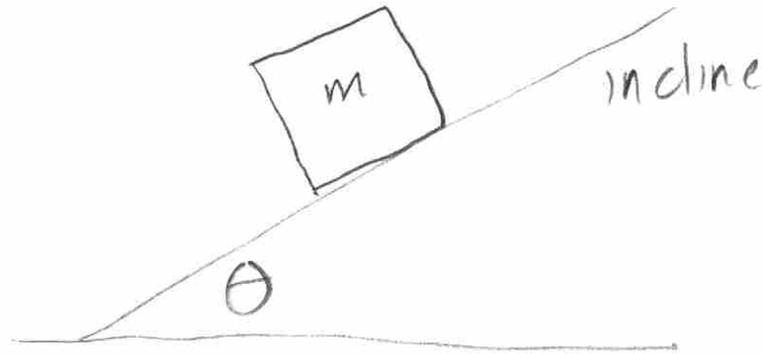
vertical:

$$N\cos\theta + f\sin\theta - mg = 0$$

horizontal

$$N\sin\theta - f\cos\theta = 0$$

To determine μ_s



increase θ until mass slides

$$|f| \leq |\mu_s N|$$

$$N - mg\cos\theta = 0$$
$$f - mg\sin\theta = 0 \quad \text{static}$$

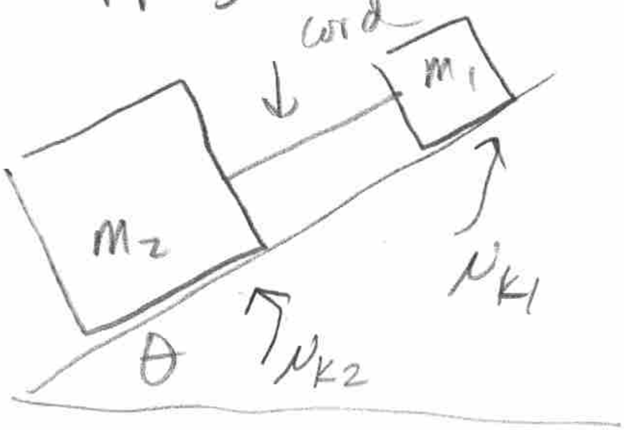
$$N = mg\cos\theta$$
$$f = mg\sin\theta$$

$$\frac{f}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \boxed{\tan\theta = \mu_s \quad \text{slippage}}$$

do demo!

AFTER SLIPPING STARTS...
accelerates...

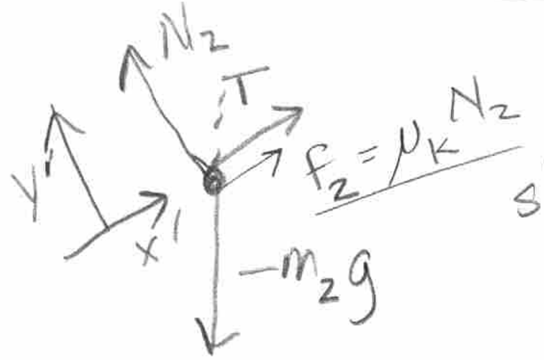
When slipping...

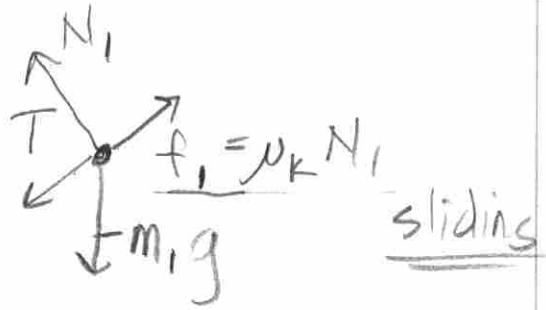


Tension in cord? Acceleration?

Assume $\boxed{\tan\theta > \mu_K}$

$\mu_K = 0$?? $\underline{a = g\sin\theta}$ BOTH!





$$N_2 - m_2 g\cos\theta = 0$$

$$N_2 = m_2 g\cos\theta$$

$$N_1 - m_1 g\cos\theta = 0$$

$$N_1 = m_1 g\cos\theta$$

$$-m_2\ddot{x}' = -m_2 g\sin\theta + \overbrace{\mu_{K2} m_2 g\cos\theta}^{\mu_{K2}N_2} + T$$

↓

$$-m_1\ddot{x}' = -m_1 g\sin\theta + \mu_{K1} m_1 g\cos\theta - T$$

$$\ddot{x}' = g\sin\theta - \mu_{K2} g\cos\theta - \frac{T}{m_2} \qquad \ddot{x}' = g\sin\theta - \mu_{K1} g\cos\theta + \frac{T}{m_1}$$

$$-\mu_{k2}\, g\cos\theta - \frac{T}{m_2} = -\mu_{k1}\, g\cos\theta + \frac{T}{m_1}$$

$$T\underbrace{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = (\mu_{k1} - \mu_{k2})\, g\cos\theta$$

reduced mass $\quad \frac{1}{\mu} \equiv \frac{1}{m_1} + \frac{1}{m_2} \qquad \mu$ .. not coefficient of friction!

$$\boxed{T = \mu(\mu_{k1} - \mu_{k2})\, g\cos\theta}$$

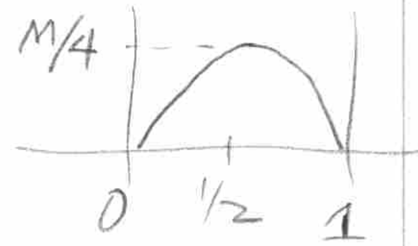
let $\quad m_1 = F M \qquad m_2 = (1-F)M \qquad$ hold M constant

vary F

$$\frac{1}{\mu} = \frac{1}{FM} + \frac{1}{(1-F)M}$$

$$= \frac{(1-F) + F}{F(1-F)M} = \frac{1}{F(1-F)M}$$

$$\mu = F(1-F)M$$



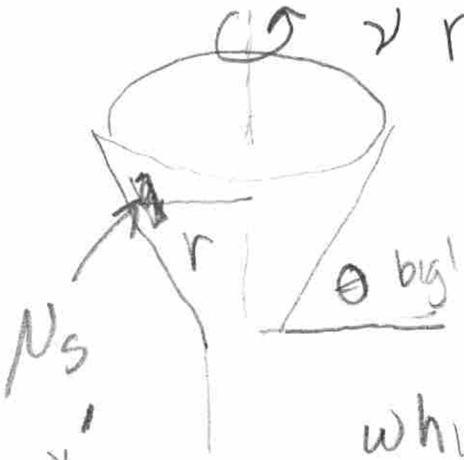
$$\ddot{x}' = g\sin\theta - \mu_{k2}\, g\cos\theta - \frac{m_1 \cancel{m_2}}{\cancel{m_2}(m_1 + m_2)}(\mu_{k1} - \mu_{k2})\, g\cos\theta$$

$$= g\sin\theta - \left(1 - \frac{m_1}{m_1 + m_2}\right)\mu_{k2}\, g\cos\theta - \frac{m_1}{m_1 + m_2}\mu_{k1}\, g\cos\theta$$

$$\boxed{\ddot{x}' = g\sin\theta - \left(\frac{m_2}{m_1 + m_2}\mu_{k2} + \frac{m_1}{m_1 + m_2}\mu_{k1}\right) g\cos\theta}$$

Tile in a funnel RHK4 6.53

ν revolutions per second $\Rightarrow 2\pi\nu \ \frac{\text{radians}}{s}$



$$\omega = 2\pi\nu$$

Find largest, smallest values of ν for which the tile stays put...

$$|a| = \omega^2 r = 4\pi^2\nu^2 r$$

a known

Tip: work in frame (axes) where unknowns need not be resolved.

y': $f - mg\sin\theta = -ma\cos\theta$

$$f = m(g\sin\theta - a\cos\theta)$$ → note meaning!

x': $N - mg\cos\theta = ma\sin\theta$

$$N = m(g\cos\theta + a\sin\theta)$$

$$\left|\frac{f}{N}\right| < \mu_s$$

$$-\mu_s < \frac{g\sin\theta - a\cos\theta}{g\cos\theta + a\sin\theta} < \mu_s$$

$$-\mu_s < \frac{g\tan\theta - a}{g + a\tan\theta} < \mu_s$$

$$-\mu_s(g + a\tan\theta) < g\tan\theta - a < \mu_s(g + a\tan\theta)$$

$$\swarrow \qquad\qquad \downarrow$$

$$a(-\mu_s\tan\theta + 1) < g(\tan\theta + \mu_s) \qquad g(\tan\theta - \mu_s) < a(1 + \mu_s\tan\theta)$$

$$a < g\,\frac{\mu_s + \tan\theta}{-\mu_s\tan\theta + 1} \qquad a > g\,\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}$$

$$g\,\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta} < 4\pi^2\nu^2 r < g\,\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}$$

$$\frac{1}{2\pi}\left[\frac{g}{r}\,\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}\right]^{1/2} < \nu < \frac{1}{2\pi}\left[\frac{g}{r}\,\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right]^{1/2}$$