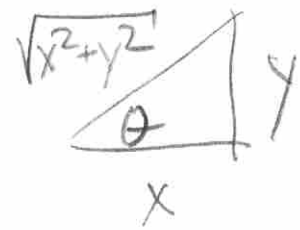
$$\vec{r} \equiv x\hat{\imath} + y\hat{\jmath}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}} = \frac{\vec{r}}{r}$$

$$\boxed{\vec{r} = r\hat{r}}$$

$\uparrow$ varies

θ gno?



$$\cos\theta = x/\sqrt{x^2+y^2}$$

$$\sin\theta = y/\sqrt{x^2+y^2}$$

$$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{\theta} = -\sin\theta\,\hat{\imath} + \underbrace{\cos\theta\,\hat{\jmath}}_{\text{points up on } x \text{ axis}}$$

$\uparrow$ $\perp$

<u>Velocity</u>: $\hat{\imath}, \hat{\jmath}$ don't change as particle moves

$\hat{r}, \hat{\theta}$ DO

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{\imath} + y\hat{\jmath}\right) = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$

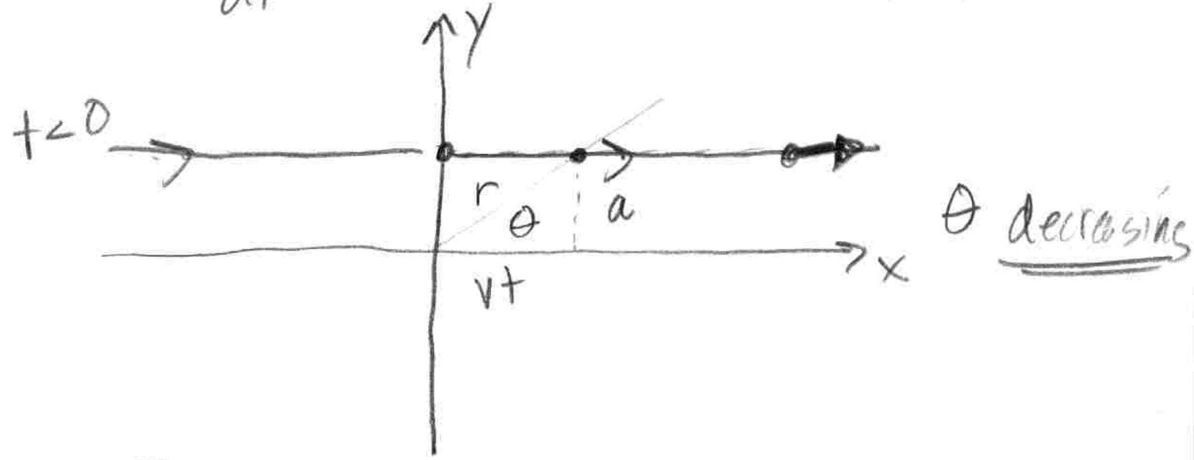
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{r}) = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

What is this saying?

Example; simple... $y = a$
$x = vt$

$$\vec{r}(t) = vt\,\hat{\imath} + a\hat{\jmath}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v\hat{\imath} \qquad \dot{x} = v\,,\ \dot{y} = 0$$



Polar? Geometric... algebraic... interplay

$$r = \sqrt{x^2+y^2} = \sqrt{(vt)^2 + a^2}$$

$$\vec{r}(t) = r\,\hat{r} = \sqrt{(vt)^2+a^2}\;\hat{r}$$

$\hat{\theta}$ never involved!

---

but: $$\dot{r} = \frac{1}{2}\left((vt)^2+a^2\right)^{-1/2} \cdot 2v^2 t$$

$$= \frac{vt}{\sqrt{(vt)^2+a^2}}\, v = v\cos\theta(t)$$

$\dot{\theta}$? $$\sin\theta = \frac{a}{\sqrt{(vt)^2+a^2}}$$

$$\cos\theta\,\dot{\theta} = -\frac{1}{\cancel{2}}\frac{a}{\left((vt)^2+a^2\right)^{3/2}}\,\cancel{2}v^2 t = \frac{-av^2t}{\left((vt)^2+a^2\right)^{3/2}}$$

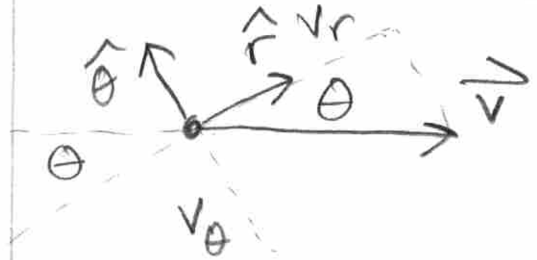
as $t \to \infty$, $\cos\theta \to 1$, $\dot\theta \to \frac{-av^2t}{v^3t^3} \to \frac{-a}{vt^2}$

look at picture, makes sense.

also $$-\frac{av^2t}{((vt)^2+a^2)^{3/2}} = -\frac{a^2}{(vt)^2+a^2}\cdot\frac{vt}{\sqrt{(vt)^2+a^2}}\cdot\frac{v}{a}$$

$$\cos\theta\,\dot\theta = -\sin^2\theta\cos\theta\,\frac{v}{a}$$

$$\dot\theta = -\sin^2\theta(t)\,\frac{v}{a}$$



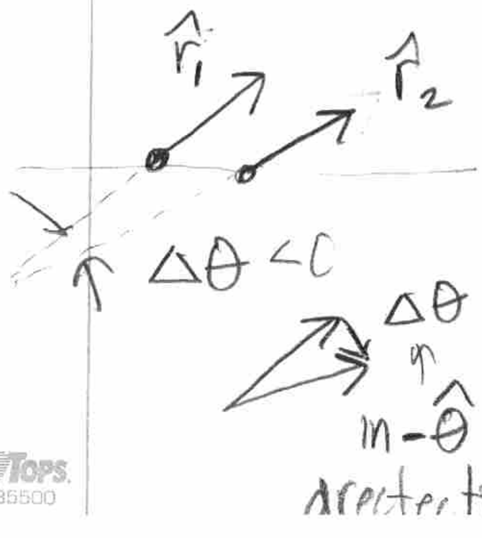
$$v_r = v\cos\theta$$

$$v_\theta = -v\sin\theta$$

$$\vec v = v_r\hat r + v_\theta\hat\theta$$

$$= \underbrace{v\cos\theta}_{\dot r}\,\hat r\ \underbrace{-\ v\sin\theta\,\hat\theta}_{\underline{\underline{\text{not}}}\ \dot\theta}$$

dimensions? originates in... $\frac{d\hat r}{dt}$



$|\hat r_1| = |\hat r_2|$, only direction changes

$\hat r_2 - \hat r_1 = \Delta\hat r$, $|\Delta\hat r| = \underline{1}\cdot\Delta\theta$

$$\Delta\hat r = \dot\theta\,\Delta t\cdot\hat\theta$$

$$\frac{\Delta\hat{r}}{\Delta t} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\boxed{\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$ — do picture (?)

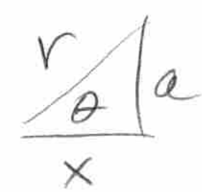
if true...

from example

$$r(\cancel{-}\sin^2\theta(t))\frac{v}{a} \stackrel{?}{=} \cancel{-v\sin\theta}\ 1$$

$$\frac{r}{a}\sin\theta \stackrel{?}{=} 1$$

$$\sin\theta \stackrel{?}{=} \frac{a}{r}$$



YES

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$ as well...

~~skip to pic~~

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\hat{\theta}}{dt}$$

for straight line above, $\vec{a} = 0$

algebra... hard!

$$\vec{a} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

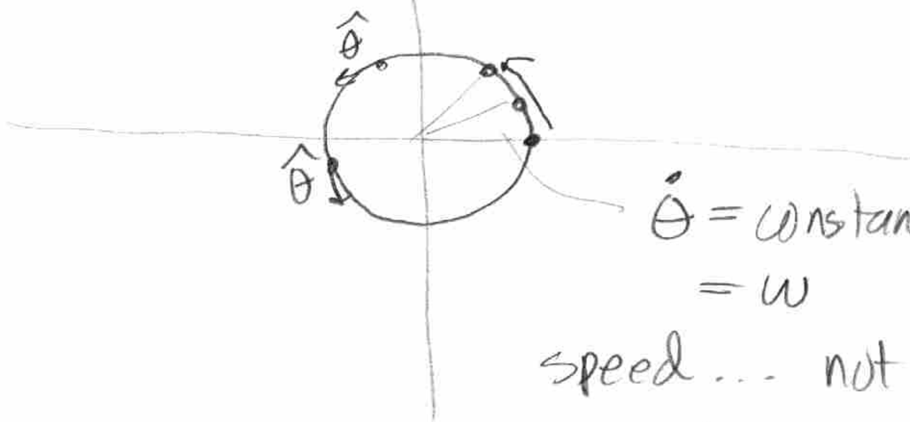
$$= (\underbrace{\ddot{r}}_{\text{easy}} - \underbrace{r\dot{\theta}^2}_{\text{centripetal}})\hat{r} + (\overbrace{r\ddot{\theta}}^{\text{easy}} + \underbrace{2\dot{r}\dot{\theta}}_{\text{coriolis.}})\hat{\theta}$$

What r,θ designed for, really ...

on a circle

$$\vec{v} = \underbrace{\dot{r}}_{0}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$= r\dot{\theta}\hat{\theta}$$



$\dot{\theta} = \text{constant}$

$= \omega$

speed ... not $\dot{\theta} = \omega$

$\Delta\theta = \dot{\theta}\Delta t$

$= \omega\Delta t$

$|\Delta\vec{r}| = r\omega\Delta t$

$$\frac{|\Delta\vec{r}|}{\Delta t} = r\omega = r\dot{\theta}$$

acceleration in $\hat{\theta}$ direction ...

$$r\ddot{\theta} = r\dot{\omega}$$

$\ddot{r}\hat{r}$ easy too!

2 New terms: centripetal: $-r\dot{\theta}^2\hat{r}$

$$= -\frac{v^2}{r}\hat{r}$$

coriolis.