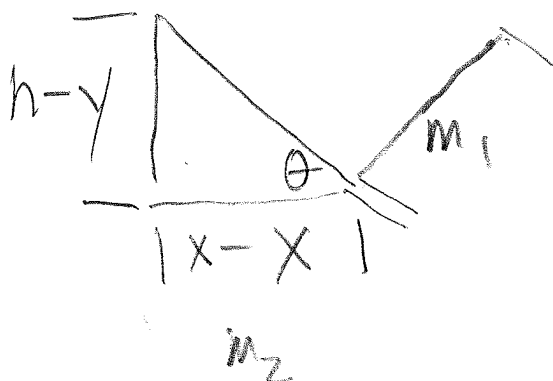


No Friction!

Coordinates

Constraint:

h = height of incline.
 y = elevation of little block.
 x = horizontal coordinate of little block
 X = horizontal coordinate of big block.

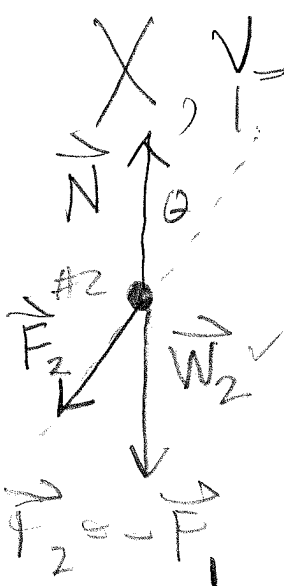


$$\tan \theta = \frac{h-y}{x-X}$$

$$(x-X) \tan \theta = h-y$$

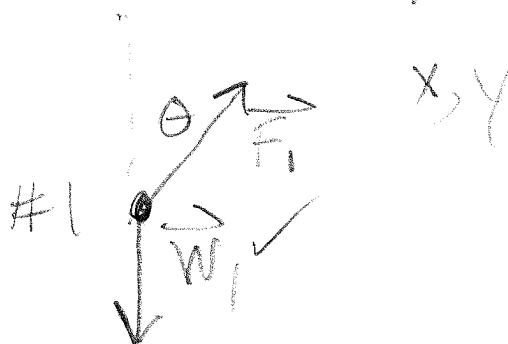
$$(\dot{x} - \dot{X}) \tan \theta = -\dot{y}$$

$$(\ddot{x} - \ddot{X}) \tan \theta = -\ddot{y}$$



$$m_2 \ddot{X} = -F_1 \sin \theta$$

$$N = N - F_1 \cos \theta - m_2 a$$



$$m_1 \ddot{x} = F_1 \sin \theta$$

$$m_1 \ddot{y} = F_1 \cos \theta - m_1 g$$

Go for $\ddot{y} \rightarrow$ eliminate all else
(like 2.16 on homework).

$$\ddot{x} = -\frac{F_1}{m_2} \sin \theta$$

$$\left(\ddot{x} + \frac{F_1}{m_2} \sin \theta \right) \tan \theta = -\ddot{y}$$

$$\ddot{x} \tan \theta = -\ddot{y} - \frac{F_1}{m_2} \sin \theta \tan \theta$$

$$\ddot{x} = \frac{-\ddot{y}}{\tan \theta} - \frac{F_1}{m_2} \sin \theta$$

$$\frac{-m_1 \ddot{y}}{\tan \theta} - \frac{m_1}{m_2} F_1 \sin \theta = F_1 \sin \theta$$

$$\frac{-m_1 \ddot{y}}{\tan \theta} = \left(1 + \frac{m_1}{m_2} \right) F_1 \sin \theta$$

$$F_1 = \frac{-m_1}{1 + \frac{m_1}{m_2}} \frac{1}{\tan \theta \sin \theta} \ddot{y}$$

$$= \underbrace{-\frac{m_2 m_1}{m_1 + m_2}}_{\text{called "reduced mass" } \mu} \frac{1}{\tan \theta \sin \theta} \ddot{y}$$

called "reduced mass" μ

μ

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

} μ simplifies the eqn

$$m_1 \ddot{y} = - \mu \frac{1}{\tan \theta \sin \theta} \ddot{y} - m_1 g$$

$$\left(m_1 + \frac{\mu}{\tan^2 \theta} \right) \ddot{y} = - m_1 g$$

$$\ddot{y} = \frac{- m_1}{m_1 + \frac{\mu}{\tan^2 \theta}} \quad g = \frac{1}{1 + \left(\frac{\mu}{m_1} \right) \frac{1}{\tan^2 \theta}} g$$

(a) $\theta = 90^\circ = \pi/2$, $\tan \theta \rightarrow \infty$
 $\ddot{y} = -g$ ✓

(b) $\theta = 0^\circ = 0$, $\tan \theta \rightarrow 0$

(c) $m_2 \rightarrow \infty$
 $\rightarrow \frac{-\tan^2 \theta}{1 + \tan^2 \theta} g \rightarrow -\sin^2 \theta g \quad \ddot{y} \rightarrow 0$

(d) $m_1 \rightarrow \infty \rightarrow -g$

$$F_1 = - \mu \frac{1}{\tan \theta \sin \theta} \ddot{y} = \frac{\mu}{\tan \theta \sin \theta} \frac{1}{1 + \left(\frac{\mu}{m_1} \right) \frac{1}{\tan^2 \theta}} g$$

(a) $\theta = 90^\circ = \pi/2$, $\tan \theta \rightarrow \infty$

$$F_1 \rightarrow \frac{\mu}{\tan \theta \sin \theta} g \rightarrow \frac{\mu g \cos \theta}{\sin \theta} \rightarrow 0!$$

(b) $\theta = 0^\circ = 0$, $\rightarrow \frac{\mu}{\tan \theta \sin \theta} g \rightarrow \frac{m_1 g}{\cos \theta} \rightarrow m_1 g$

$$\ddot{X} = -\frac{F_1}{m_2} \sin \theta$$

$$= \frac{-\left(\frac{\mu}{m_2}\right) \frac{1}{\tan \theta} \sin \theta \cdot g}{1 + \left(\frac{\mu}{m_1}\right) \frac{1}{\tan^2 \theta}} \quad g$$

$$\ddot{X} = \frac{-\left(\frac{\mu}{m_2}\right) \frac{1}{\tan \theta}}{1 + \left(\frac{\mu}{m_1}\right) \frac{1}{\tan^2 \theta}} \quad g$$

$\theta \rightarrow 90^\circ = \pi/2, \tan \theta \rightarrow \infty$

$$\ddot{X} \rightarrow -\frac{\mu}{m_2} \frac{1}{\tan \theta} g \rightarrow 0$$

$\theta \rightarrow 0^\circ \rightarrow 0$

$$\ddot{X} \rightarrow \frac{-\frac{\mu}{m_2} \frac{1}{\tan \theta}}{\frac{\mu}{m_1} \frac{1}{\tan^2 \theta}} \quad g$$

$$\rightarrow -\frac{m_1}{m_2} \tan \theta g \rightarrow 0$$

WHEN IS \ddot{X} MAXIMUM?