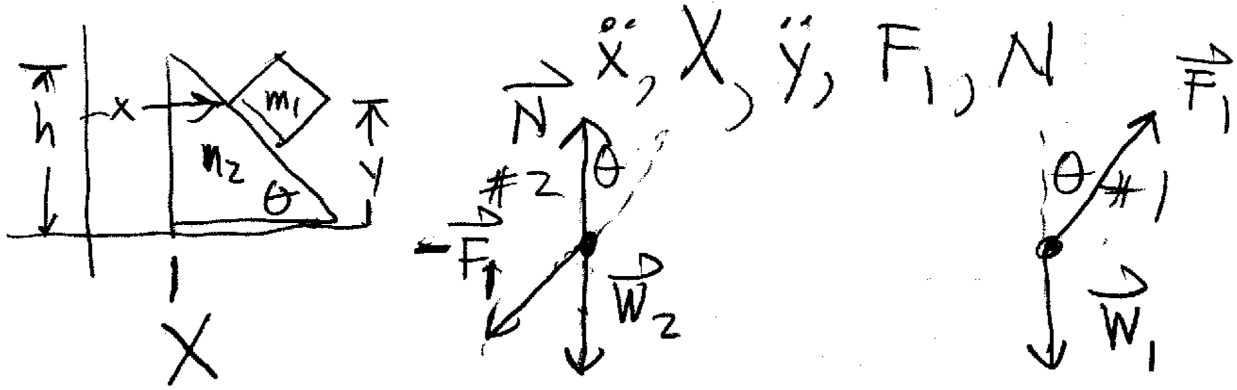
The 5 unknowns: $\ddot{x}, \ddot{X}, \ddot{y}, F_1, N$



5 equations $(\ddot{x}-\ddot{X})\tan\theta = -\ddot{y}$ (constraint)

$$m_2\ddot{X} = -F_1\sin\theta \qquad m_1\ddot{x} = F_1\sin\theta$$

$$0 = N - F_1\cos\theta - m_2 g \qquad m_1\ddot{y} = F_1\cos\theta - m_1 g$$

Kind of hard!.... let $m_2 \to \infty$ first....

then $\ddot{X} = 0$!

$$\left.\begin{aligned} \ddot{x}\tan\theta &= -\ddot{y} \\ m_1\ddot{x} &= F_1\sin\theta \\ m_1\ddot{y} &= F_1\cos\theta - m_1 g \end{aligned}\right\} \text{go for } \ddot{y}\text{, elim others!}$$

$$\ddot{x} = -\frac{\ddot{y}}{\tan\theta}$$

$$-m_1 \frac{\ddot{y}}{\tan\theta} = F_1 \sin\theta$$

$$F_1 = -m_1 \frac{\ddot{y}}{\tan\theta \sin\theta}$$

$$\cancel{m_1}\ddot{y} = -\cancel{m_1}\frac{\ddot{y}}{\tan\theta \sin\theta}\cos\theta - \cancel{m_1} g$$

m₁ cancels!

$$\ddot{y}\underbrace{\left(1 + \frac{1}{\tan^2\theta}\right)} = -g$$

$$\frac{1+\tan^2\theta}{\tan^2\theta} = \frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{\overset{1}{\cancel{\sin^2\theta+\cos^2\theta}}}{\cancel{\cos^2\theta}}}{\frac{\sin^2\theta}{\cancel{\cos^2\theta}}}$$

$$= \frac{1}{\sin^2\theta}$$

$$\ddot{y}\,\frac{1}{\sin^2\theta} = -g$$

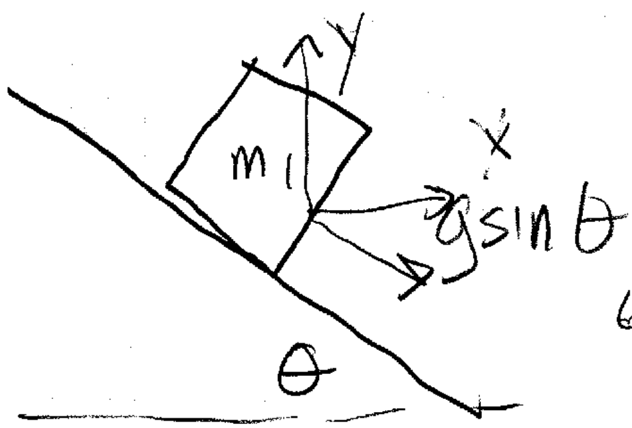
$$\boxed{\ddot{y} = -g\sin^2\theta} \qquad \ddot{x} = \frac{g \cdot \sin^2\theta}{\tan\theta}$$

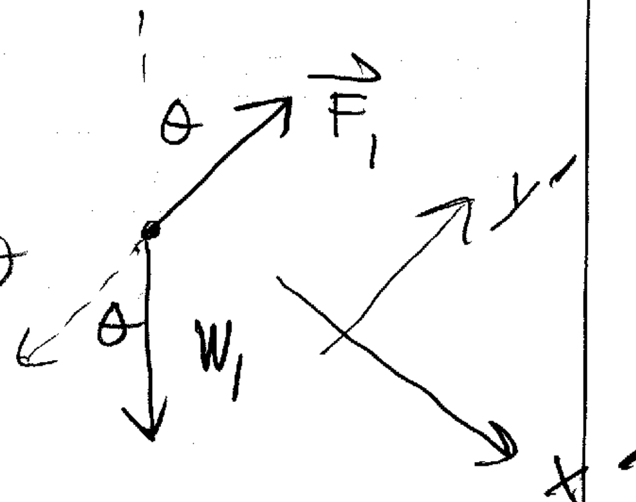
$$= \frac{g\sin^2\theta}{(\sin\theta/\cos\theta)}$$

$$\boxed{\ddot{x} = g\sin\theta\cos\theta}$$

$$\boxed{F_1 = -m_1 \frac{(-g\sin^2\theta)}{\frac{\sin\theta}{\cos\theta} \cdot \sin\theta} = m_1 g\cos\theta}$$

Pretty hard... easy way to see!





$$F_1 - m_1 g\cos\theta = 0$$

$$F_1 = m_1 g\cos\theta$$

$$\cancel{m_1}\ddot{x}_1' = \cancel{m_1} g\sin\theta$$

$$\ddot{x}_1 = \ddot{x}'\cos\theta = g\sin\theta\cos\theta$$

$$\ddot{y} = -\ddot{x}'\sin\theta = -g\sin^2\theta$$

Go for $\ddot{y}$ → eliminate all else
(like 2.16 on homework).

$$\ddot{X} = -\frac{F_1}{m_2}\sin\theta$$

$$\left(\ddot{x} + \frac{F_1}{m_2}\sin\theta\right)\tan\theta = -\ddot{y}$$

$$\ddot{x}\tan\theta = -\ddot{y} - \frac{F_1}{m_2}\sin\theta\tan\theta$$

$$\ddot{x} = \frac{-\ddot{y}}{\tan\theta} - \frac{F_1}{m_2}\sin\theta$$

$$\frac{-m_1\ddot{y}}{\tan\theta} - \frac{m_1}{m_2}F_1\sin\theta = F_1\sin\theta$$

$$\frac{-m_1\ddot{y}}{\tan\theta} = \left(1+\frac{m_1}{m_2}\right)F_1\sin\theta$$

$$F_1 = -\frac{m_1}{1+\frac{m_1}{m_2}}\,\frac{1}{\tan\theta\sin\theta}\,\ddot{y}$$

$$= -\underbrace{\frac{m_2 m_1}{m_1+m_2}}_{\text{called "reduced mass"}}\,\frac{1}{\tan\theta\sin\theta}\,\ddot{y}$$

μ

$$\left.\begin{aligned}\mu &= \frac{m_1 m_2}{m_1+m_2}\\ \frac{1}{\mu} &= \frac{1}{m_1}+\frac{1}{m_2}\end{aligned}\right\} \ \mu \text{ smaller than either}$$

$$\ddot{x} = \frac{-\ddot{y}}{\tan\theta} - \frac{F_1}{m_2}\sin\theta$$

$$= \left(\frac{+1}{\tan\theta + \left(\frac{\mu}{m_1}\right)\frac{1}{\tan\theta}} - \frac{\left(\frac{\mu}{m_2}\right)\frac{1}{\tan\theta}}{1 + \left(\frac{\mu}{m_1}\right)\frac{1}{\tan^2\theta}} \right) g$$

$$= \frac{1 - \frac{\mu}{m_2}}{\tan\theta + \left(\frac{\mu}{m_1}\right)\frac{1}{\tan\theta}}$$

$$1 - \frac{\mu}{m_2} = 1 - \frac{m_1}{m_1+m_2} = \frac{m_2}{m_1+m_2} = \frac{\mu}{m_1}$$

$$\boxed{\ddot{x} = \frac{(\mu/m_1)\tan\theta}{(\mu/m_1) + \tan^2\theta}\, g}$$

$m_2 \to \infty$, $\frac{\mu}{m_1} \to 1$

$$\ddot{x} \to \frac{\tan\theta}{1+\tan^2\theta} g = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta \to 1}{\cos^2\theta}} g$$

$$\ddot{x} \to \sin\theta\cos\theta \cdot g$$