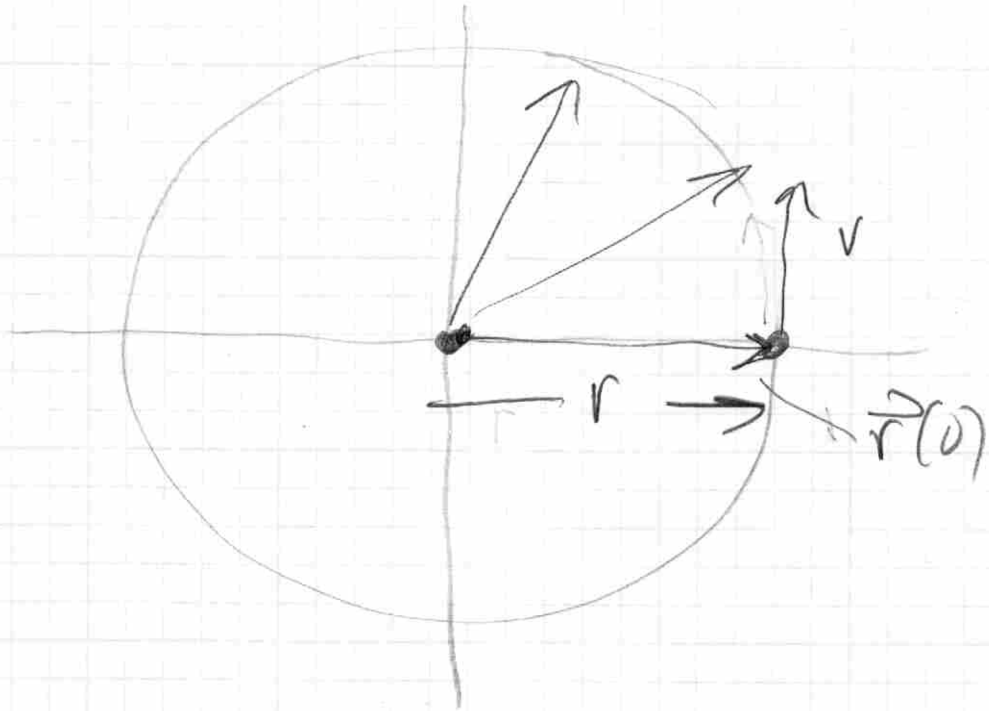
More circular motion

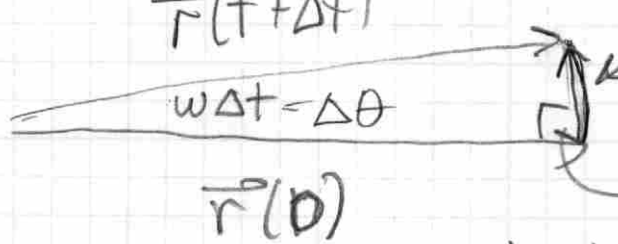


make $\vec{r}(t) = r\cos(\omega t)\hat{i} + r\sin(\omega t)\hat{j}$

$t=0 \quad \vec{r}(0) = r\hat{i}$

$\vec{v}(t) = r\omega\left[-\sin(\omega t)\hat{i} + \cos(\omega t)\hat{j}\right]$

another way to reason this out -
tiny time Δt



$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$

as $\Delta t \to 0$

angle $\to 90°$

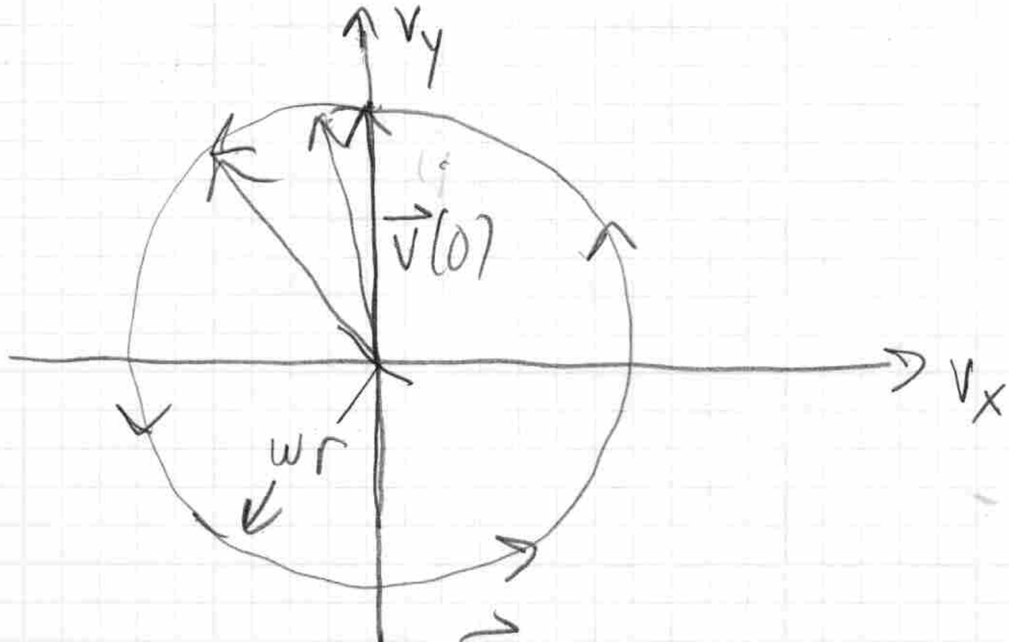
length: $|\Delta\vec{r}| \simeq \Delta\theta \cdot r$

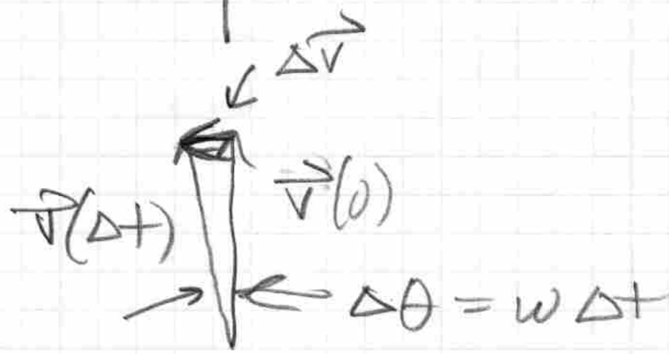
$$\text{length} = r\Delta\theta = r\omega\Delta t$$

$$|\vec{v}| = \frac{r\omega\Delta t}{\Delta t} = r\omega = v$$

$$\omega = \frac{v}{r} \qquad \frac{[l/t]}{[l]} = \frac{1}{t}$$

OK, now visualize $v(t)$





$$|\Delta\vec{v}| \cong \Delta\theta \cdot |\vec{v}| = \omega\Delta t \cdot \omega r$$

$$\frac{|\Delta\vec{v}|}{\Delta t} = \omega^2 r$$

$\vec{a}$ is non-zero, points right to left, opposite to $\vec{r}$

$$|\vec{a}| = \omega^2 r = \frac{v^2}{r^2} \cdot r = \frac{v^2}{r}$$

Algebraic:

$$\vec{v}(t) = r\omega\left[-\sin(\omega t)\,\hat{\imath} + \cos(\omega t)\,\hat{\jmath}\right]$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = r\omega\left[-\sin(\omega t)\,\omega\,\hat{\imath} - \cos(\omega t)\,\omega\,\hat{\jmath}\right]$$

$$= -r^2\omega\,\underbrace{\left[\sin(\omega t)\,\hat{\imath} + \cos(\omega t)\,\hat{\jmath}\right]}_{\parallel \text{ to } \vec{r}}$$

$\parallel$ to $\vec{r}$

$-$ sign

magnitude is $r^2\omega$.

Newton's Laws:

Read KK 52–75 (not so hard)..

$\boxed{N1}$ Isolated bodies maintain $\underbrace{\text{constant velocity}}_{\vec{v}=0 \text{ best}}$ in absence of net external forces.

$\boxed{N2}$ In presence of a net external force $\vec{F}_{net}$, $m\vec{a} = \vec{F}_{net}$.

$[F] = m \cdot \ell / s^2$: m is <u>mass</u> in kg $\neq$ <u>weight</u> !

In outer space you still have mass, but no weight!

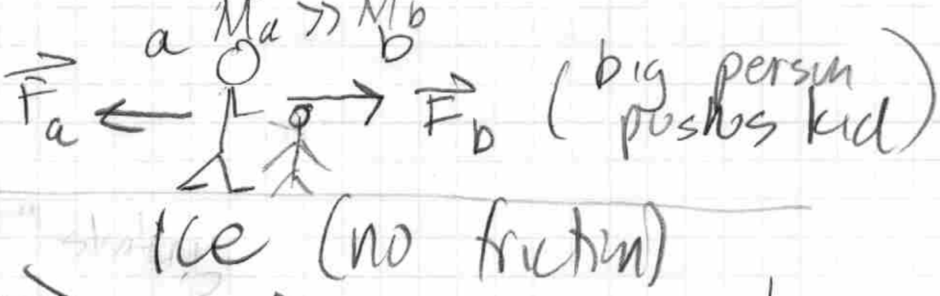
mg is weight

m is mass.

$m \propto$ # protons + # neutrons

$$\frac{kg \cdot meter}{s^2} \equiv \text{NEWTON}.$$

$\boxed{N3}$ To every action (force applied) there is an equal and opposite (in a direction) reaction. (WEIRDEST)



$\vec{F}_a = -\vec{F}_b$ inevitable!

$$\vec{a}_a = \frac{-\vec{F}_a}{M_a} \qquad \vec{a}_b = \frac{\vec{F}_a}{M_b} \qquad \frac{|\vec{a}_a|}{|\vec{a}_b|} = \frac{\frac{1}{M_a}}{\frac{1}{M_b}} = \frac{M_b}{M_a}$$