Into the second + third dimensions!

Constant acceleration: 1-d

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

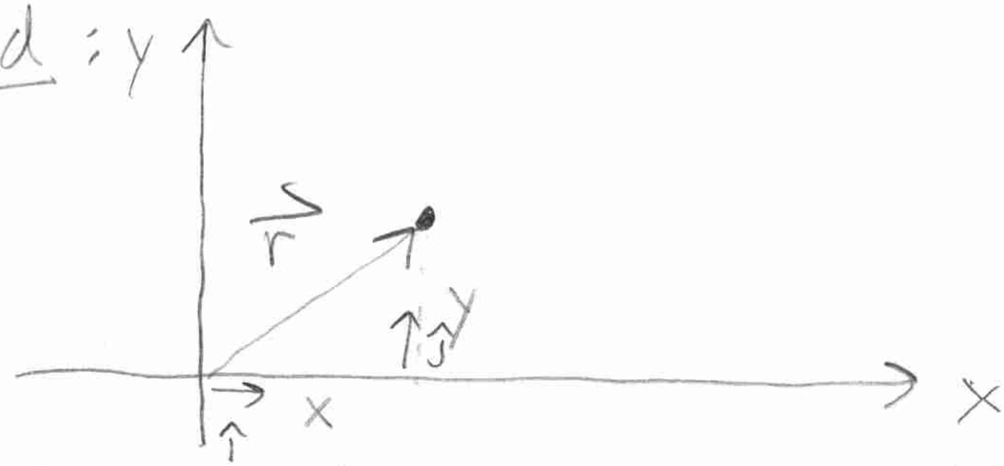
$x = \frac{1}{2} a t^2 + v_0 t + x_0$

$\dot{x} = v = \frac{dx}{dt} = v_0 + at$

$\frac{dx}{dt} = \int a\,dt = at + v_0$

$\ddot{x} = a = \frac{d^2x}{dt^2} = a$

<u>2-d</u>:



locations of $\hat{i}$ + $\hat{j}$ don't matter (free vector)

$\vec{r} = x\hat{i} + y\hat{j}$ ← DEPENDS ON ORIGIN'S LOCATION!

TIP: look at $t=0$, $x(0)$, $y(0)$, $[z(0)]$

⇒ PUT ORIGIN THERE

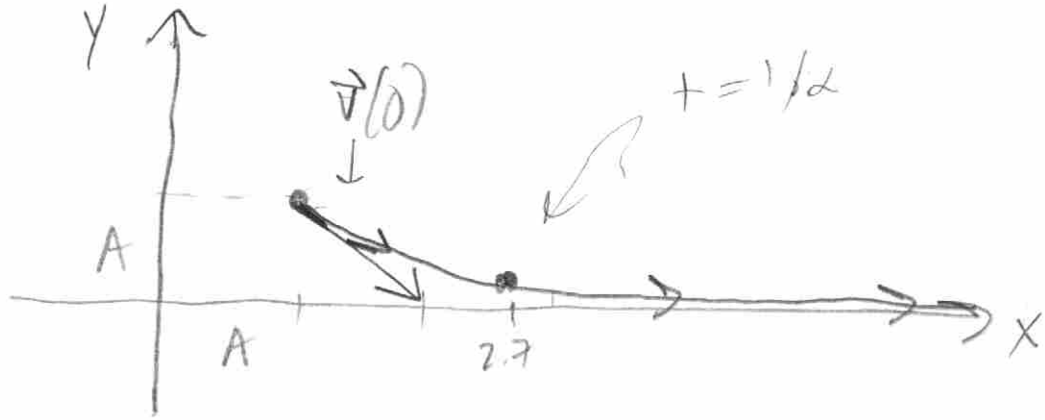
so, choose $\vec{r}(0) = 0$ when possible

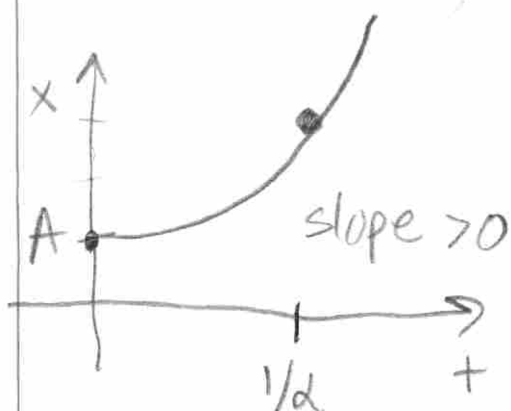
SUBSEQUENT MOTION: $x(t)$, $y(t)$

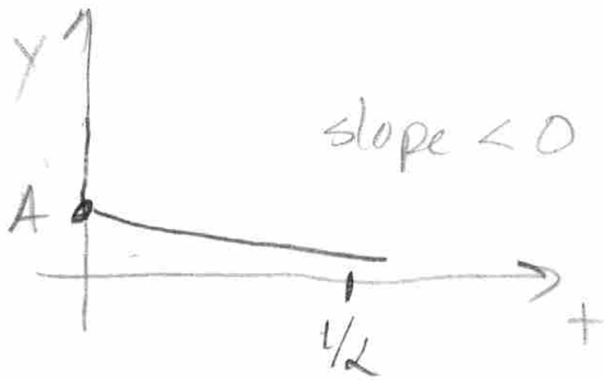
Hard to visulize.. Example 1.7 KK

$x(t) = A e^{\alpha t}$ $\qquad$ $y(t) = A e^{-\alpha t}$

$t = 0$ $\qquad$ $x(0) = A$ $\qquad$ $y(0) = A$







$[A] = l$

$[\alpha] = \frac{1}{t}$

when $t = 1/\alpha$, $x(\frac{1}{\alpha}) = A \cdot \underset{\substack{\uparrow \\ 2.7}}{e}$

$\uparrow$ wins in the end

$\vec{v} = ?$ $\qquad$ $\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} = A(\overbrace{e^{\alpha t}}\hat{\imath} + e^{-\alpha t}\hat{\jmath})$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + x\cancelto{0}{\frac{d\hat{\imath}}{dt}} + \frac{dy}{dt}\hat{\jmath} + y\cancelto{0}{\frac{d\hat{\jmath}}{dt}}$$

$$\frac{d\vec{r}}{dt} = v_x \hat{i} + v_y \hat{j}$$

look at dimensions $[A\alpha] = \ell/t$

$$v_x = \frac{d}{dt}\left(Ae^{\alpha t}\right) = A\alpha e^{\alpha t} > 0 \text{ (remember)}$$

$$v_y = \frac{d}{dt}\left(Ae^{-\alpha t}\right) = -A\alpha e^{-\alpha t} < 0 \; !$$

$$\vec{v}(t) = A\alpha e^{\alpha t}\hat{i} - A\alpha e^{-\alpha t}\hat{j}$$

$$= A\alpha\left(e^{\alpha t}\hat{i} - e^{-\alpha t}\hat{j}\right)$$

downward

$t=0$, $v(0) = \hat{i} - \hat{j}$

NOW SPEED HAS NEW MEANING

$= |\vec{v}|$ (was "trivial" in 1-d).

$$= \sqrt{v_x^2 + v_y^2} = \sqrt{A^2\alpha^2 e^{2\alpha t} + A^2\alpha^2\left(-e^{-\alpha t}\right)^2}$$

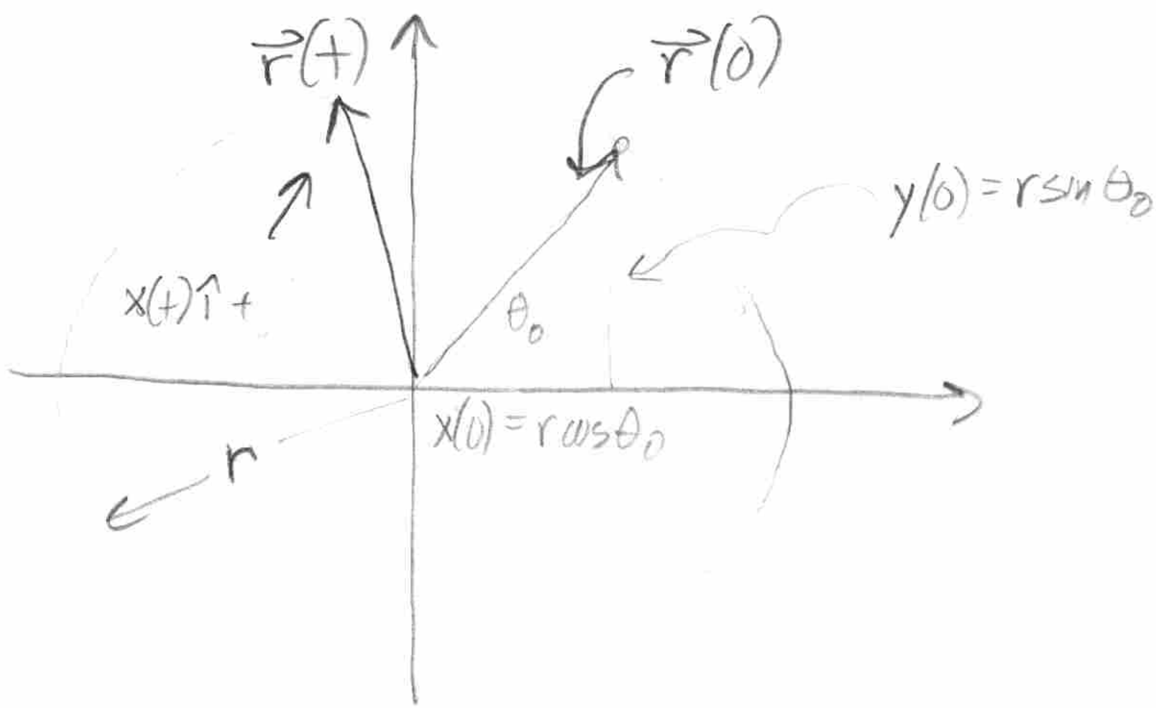
$$= A\alpha\sqrt{e^{2\alpha t} + e^{-2\alpha t}}$$

$t=0 \quad = \sqrt{2}\,A\alpha$

$t \to \infty \quad \to A\alpha e^{\alpha t} \to \infty$

$$\vec{a}(t) = A\alpha^2\left(e^{\alpha t}\hat{i} + e^{-\alpha t}\hat{j}\right)$$

Circular Motion Radius r



① Where on the circle does the mass start? $0 < \theta_0 < 2\pi$

or $-\pi < \theta_0 < \pi$

$$x(0) = r\cos\theta_0 \qquad \vec{r}(0) = r\cos\theta_0\,\hat{\imath} + r\sin\theta_0\,\hat{\jmath}$$
$$y(0) = r\sin\theta_0$$

② $\theta = \theta_0 + \omega t$ uniform circular motion

θ: radians

ω: constant called "angular velocity"

$$[\omega] = \frac{\text{radians}}{\text{second}}$$

③ $x(t) = r\cos(\theta_0 + \omega t) \qquad y(t) = r\sin(\theta_0 + \omega t)$

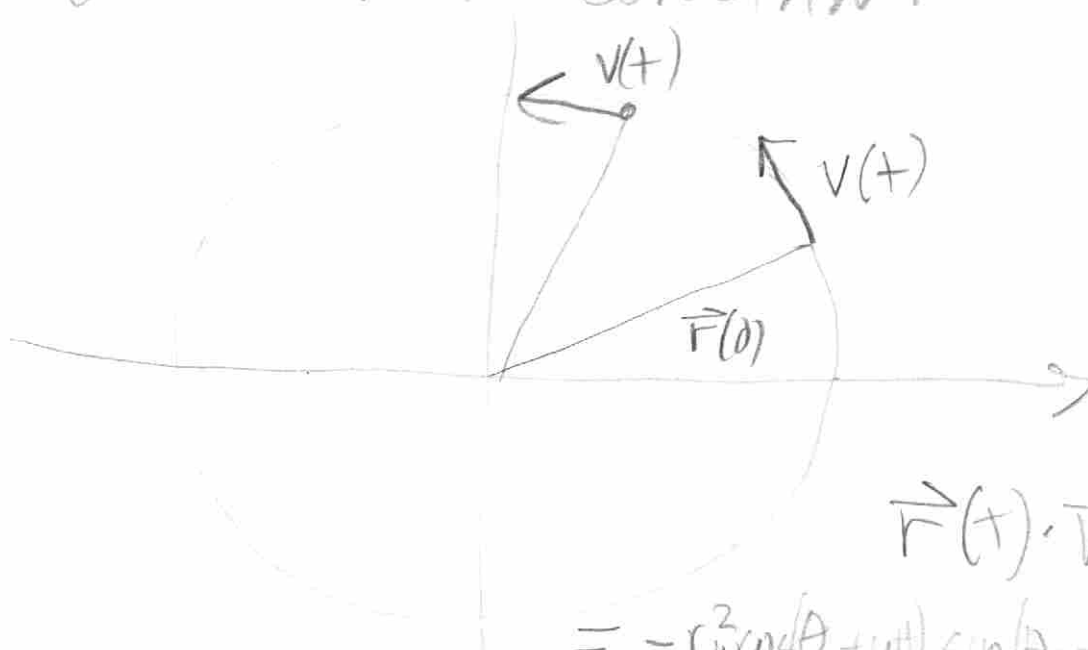
$$\vec{r}(t) = r\cos(\theta_0+\omega t)\,\hat{i} + r\sin(\theta_0+\omega t)\,\hat{j}$$
$$= r\left(\cos(\theta_0+\omega t)\,\hat{i} + \sin(\theta_0+\omega t)\,\hat{j}\right)$$
$$\vec{v}(t) = -r\sin(\theta_0+\omega t)\,\omega\,\hat{i} + r\cos(\theta_0+\omega t)\,\omega\,\hat{j}$$
$$= r\omega\left[-\cos(\theta_0+\omega t)\,\hat{i} + \sin(\theta_0+\omega t)\,\hat{j}\right]$$

$$|\vec{r}(t)| = \sqrt{r^2\cos^2(\theta_0+\omega t) + r^2\sin^2(\theta_0+\omega t)}$$
$$= r\sqrt{\underbrace{\cos^2(\theta_0+\omega t) + \sin^2(\theta_0+\omega t)}_{1}}$$

$|\vec{r}(t)| = r$ constant

$|v(t)| = r\omega$ constant speed

VELOCITY NOT CONSTANT



$$\vec{r}(t)\cdot\vec{v}(t)$$
$$= -r^2\omega\cos(\theta_0+\omega t)\sin(\theta_0+\omega t)$$
$$+ r^2\omega\cos(\theta_0+\omega t)\ldots = 0$$

Projectile Motion

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

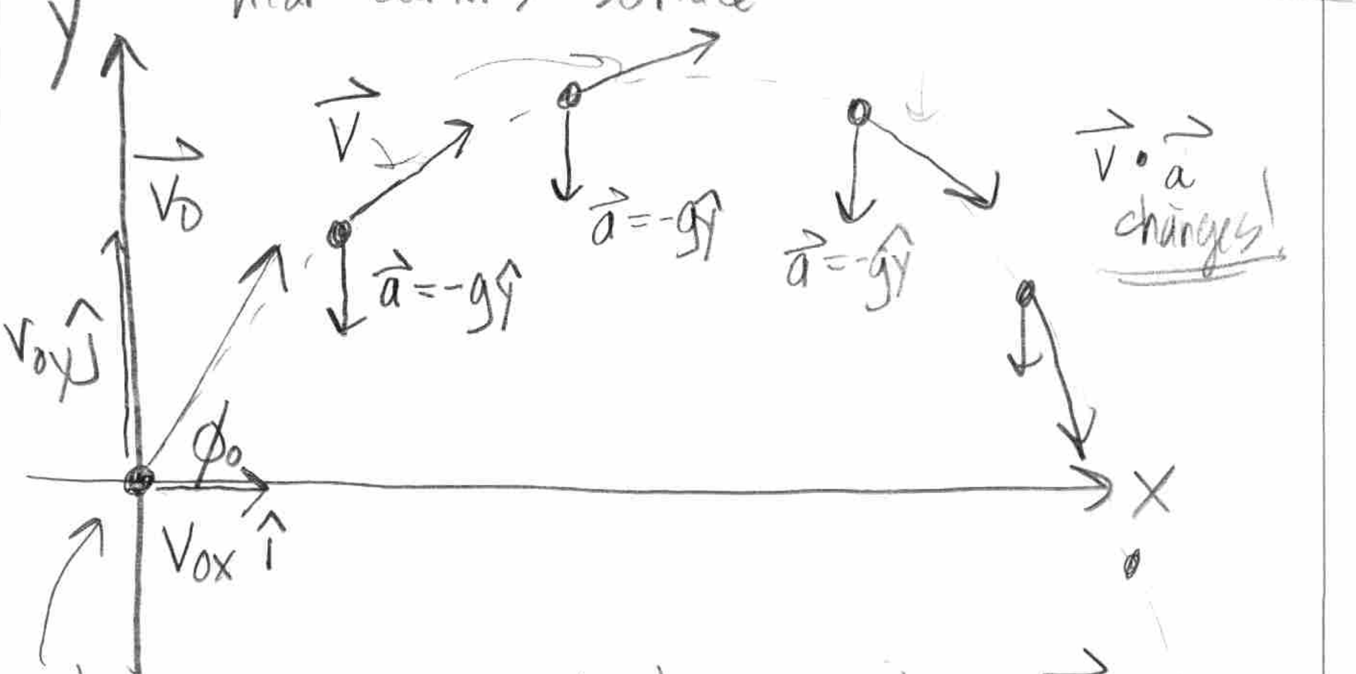
$$= (v_{0x}\hat{\imath} + v_{0y}\hat{\jmath} + v_{0z}\hat{k}) +$$

$$a_x t\hat{\imath} + a_y t\hat{\jmath} + a_z t\hat{k}$$

$\vec{a}$ & $\vec{v}$ NEED NOT BE PARALLEL.

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \quad \text{(1 problem)}$$

near earth's surface



put x-y origin on starting position, $\underline{\underline{\vec{r}_0 = 0}}$

t: independent variable, implicit

$V_0 = |\vec{V}_0|$

$V_{0x} = V_0 \cos\phi_0 \quad V_{0y} = V_0 \sin\phi_0$

NO ACCELERATION IN X-direction!

$$x = V_{0x} t = V_0 \cos\phi_0 \, t$$

ELIMINATE t: $t = \dfrac{x}{V_0 \cos\phi_0}$

$$y = V_{0y} t - \frac{1}{2} g t^2$$

$$= t\left(V_{0y} - \frac{1}{2} g t\right)$$

$$y = \frac{x}{V_0 \cos\phi_0}\left(V_0 \sin\phi_0 - \frac{1}{2} g \frac{x}{V_0 \cos\phi_0}\right)$$

time

Highest: (i) $V_y = 0$

$$= V_{0y} - gt = 0$$

$$t = \frac{V_{0y}}{g} = \frac{V_0 \sin\phi_0}{g} \qquad \frac{\ell/t}{\ell/t^2} = t$$

$$y_{max} = \frac{V_{0y}^2}{g} - \frac{1}{2} g \frac{V_{0y}^2}{g^2}$$

$$= \frac{1}{2} \frac{V_{0y}^2}{g} = \frac{1}{2} \frac{V_0^2 \sin^2\phi_0}{g} \qquad \frac{\ell^2/t^2}{\ell/t^2} = \ell$$

TOPS.

(ii) $\frac{dy}{dx} = 0 = \frac{\sin\phi_0}{\cos\phi_0} - g\frac{x}{v_0^2\cos^2\phi_0} = 0$

$$x = \frac{v_0^2}{g}\sin\phi_0\cos\phi_0$$

$$t = \frac{x}{v_0\cos\phi_0} = \frac{v_0}{g}\sin\phi_0 \checkmark$$

Range R

(i) 2 times x above

$$\boxed{R = 2\frac{v_0^2}{g}\sin\phi_0\cos\phi_0 = \frac{v_0^2}{g}\sin(2\phi_0)}$$

win as v_0^2

max $\phi_0 = 45°$ fix v_0

6 X farther on moon!

(ii) $y = 0$

$$v_0\sin\phi_0 - \frac{1}{2}g\frac{R}{v_0\cos\phi_0} = 0$$

$$R = 2\frac{v_0^2}{g}\sin\phi_0\cos\phi_0$$

LOTS OF PROBLEMS

SPS Meetre