## Constant Velocity

$x = A + Bt$

$[x] = \ell$

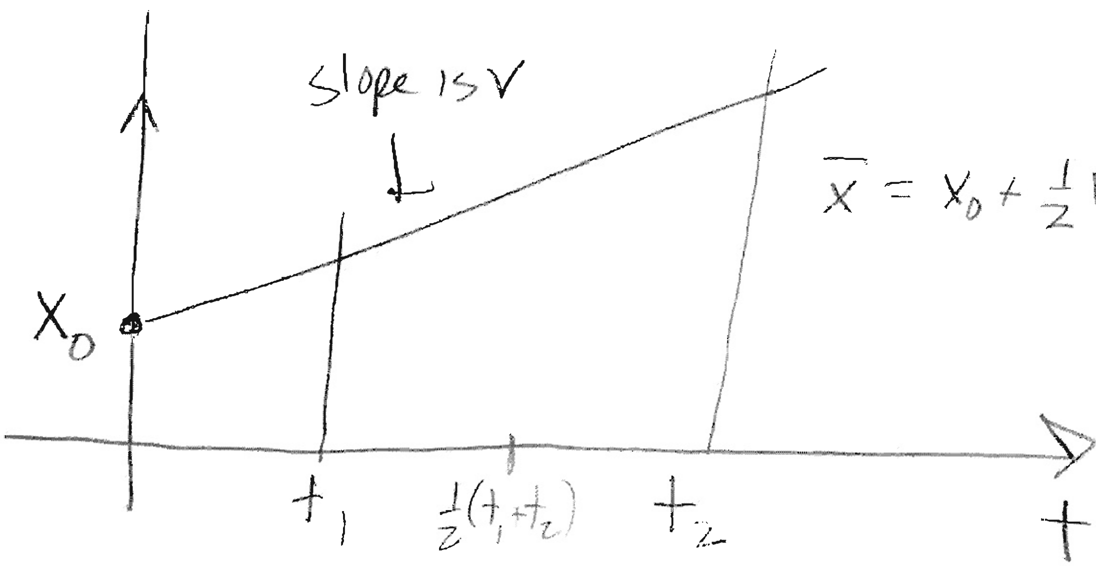
$A$: position when $t = 0$, aka $x_0$

$[A] = \ell$ (length)

$[B] = \dfrac{\ell}{t}$ like m/s

B aka v

$$x = x_0 + vt$$



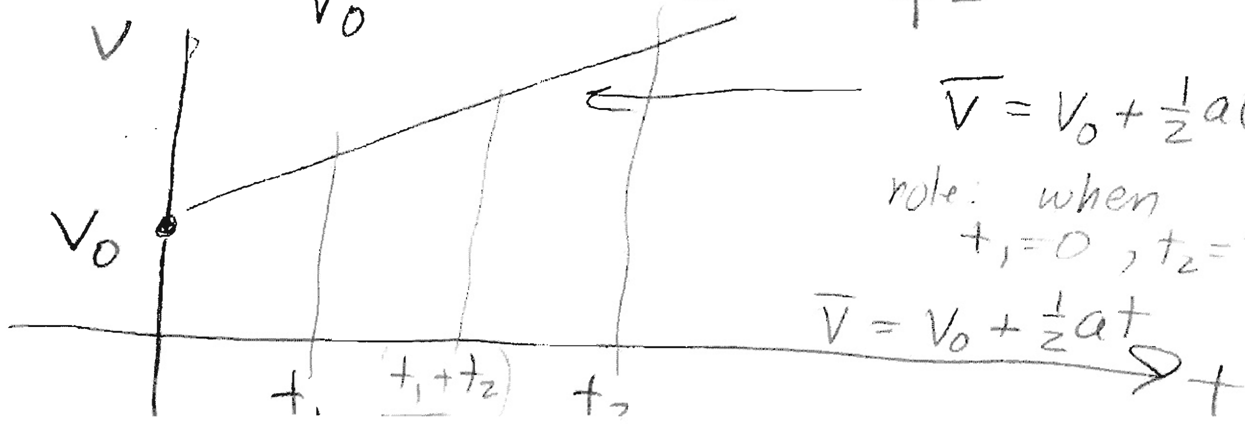

$\bar{x} = x_0 + \frac{1}{2}v(t_1+t_2)$

## Constant Acceleration

$v = B + Ct = v_0 + at$

$B$: $v_0$

$[C] = \dfrac{\ell}{t^2}$



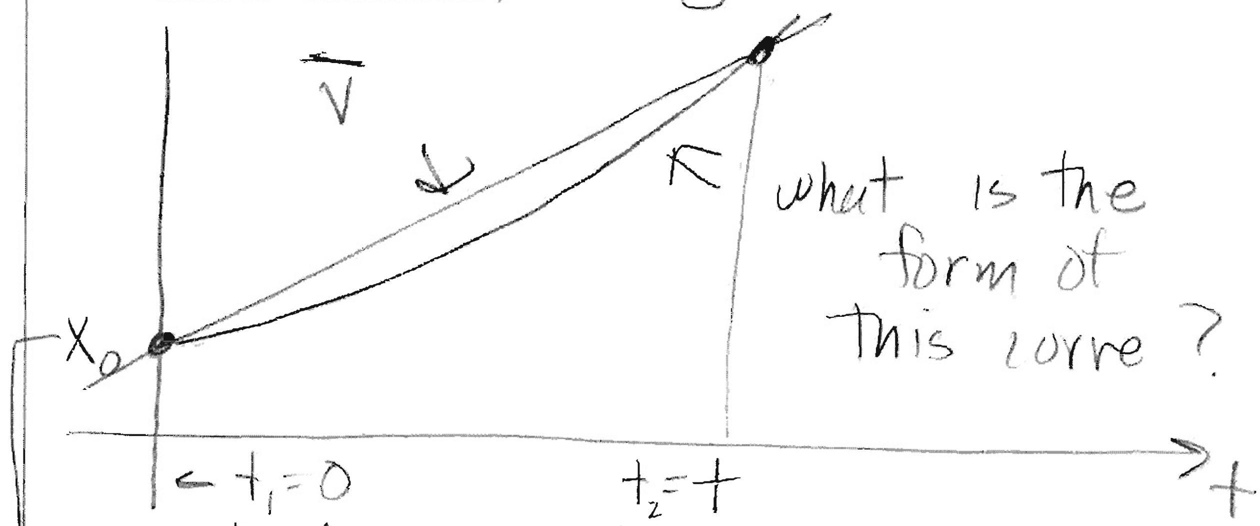

$\bar{v} = v_0 + \frac{1}{2}a(t_1+t_2)$

note: when $t_1 = 0$, $t_2 = t$

$\bar{v} = v_0 + \frac{1}{2}at$

Want $x(t)$...

Method #1, "algebraic"




starting position is not determined by velocity

$$x = x_0 + \bar{v}t$$

$$= x_0 + (v_0 + \tfrac{1}{2}at)t$$

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

$$\dot{x} = v = \frac{dx}{dt} = v_0 + 2 \cdot \tfrac{1}{2}at = v_0 + at$$

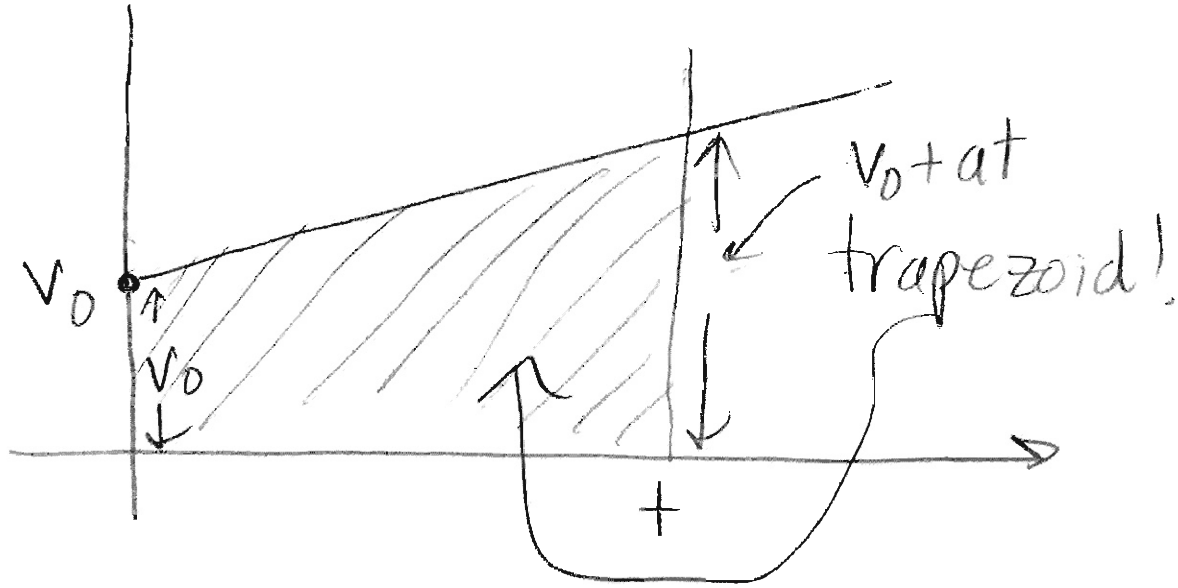
$$\ddot{x} = a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = a$$

Method #2, "geometric"

Displacement is area under the velocity curve.

Ⓐ "go with it"

When constant acceleration

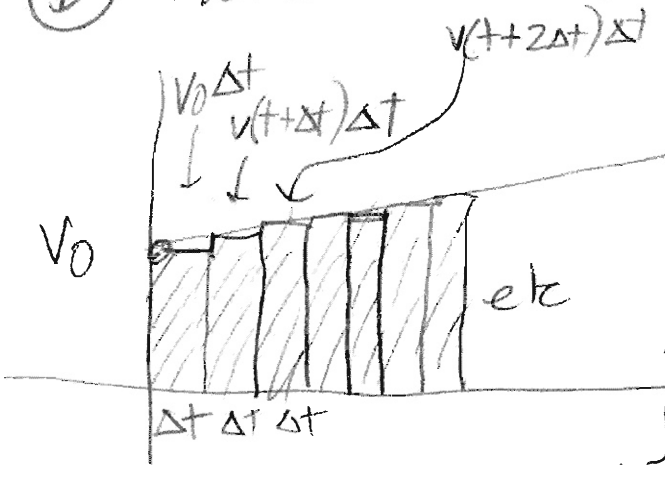



$$\text{Area} = \frac{1}{2}(v_0 + v_0 + at) \times t$$
$$= v_0 t + \frac{1}{2}at^2$$

Position = (initial position) + displacement

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

Ⓑ more detailed.




$$\Sigma \underbrace{v(t)\Delta t}_{\text{small displacements}} = \text{Area}$$

$$\int v \, dt = \text{displacement}$$

homework: $x(t) = A + Bt + Ct^2 + Dt^3$

$$v(t) = \dot{x}(t) = B + 2Ct + 3Dt^2$$

$$a(t) = \ddot{x}(t) = 2C + 6Dt$$

When $a(t) =$ constant $(D = 0!)$

can combine $v = v_0 + at$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

to eliminate $t$... what good.. your last problem PS#2

$$t = \frac{v - v_0}{a}$$

$$\underbrace{(x - x_0)}_{\text{displacement}} = v_0\left(\frac{v-v_0}{a}\right) + \frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2$$

$$= \left(\frac{v-v_0}{a}\right)\left(v_0 + \frac{1}{2}(v - v_0)\right)$$

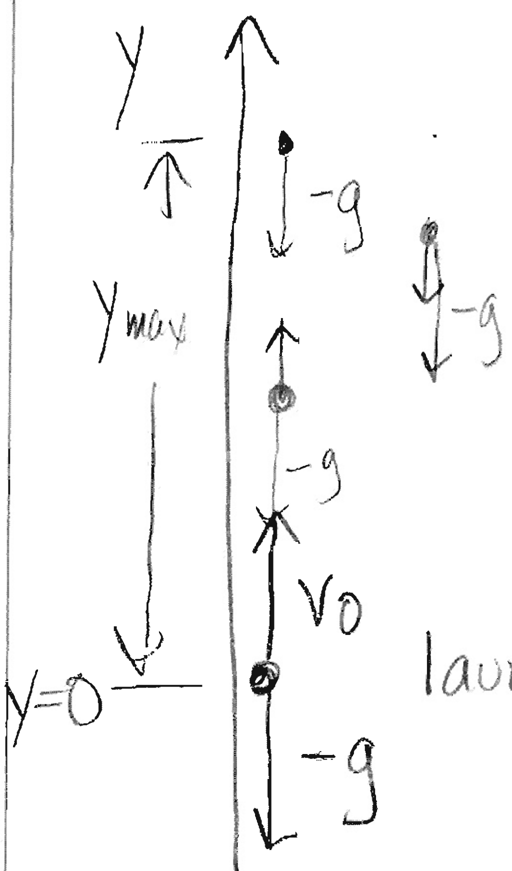
$$(x - x_0) = \frac{1}{2}\frac{1}{a}(v - v_0)(v + v_0)$$

$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

{ "potential energy

<u>Gravity</u>: near earth's surface, makes a uniform <u>downward</u> acceleration of $a = -g$

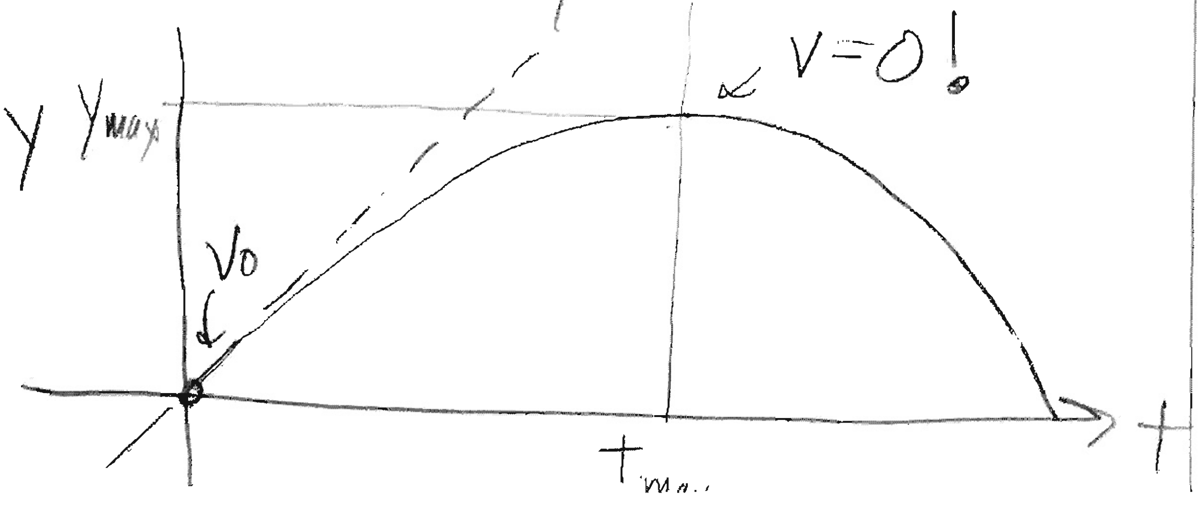
$g \cong 9.8\ m/s^2$
($10\ m/s^2$ often OK!)




Measure velocity with h

launch a ball with $v_0$ <u>upward</u>

suppose it rises to height $y_{max}$ (above release position!) .. what was initial velocity.



$$\cancel{v^2}^{\,0} - v_0^2 = 2(-g)(y_{max} - 0)$$

$$v_0 = \sqrt{2 g\, y_{max}}$$

$y_{max} = 5\ m$, say.

$$v_0 \simeq \sqrt{2 \cdot 10 \frac{m}{s^2} \cdot 5\, m} = \sqrt{100 \frac{m^2}{s^2}}$$

$$\boxed{v_0 \simeq 10\ m/s} \qquad 1\ m/s = 2.24\ mph$$

$$v_0 \simeq 22\ mph$$

Suppose $v_0 = 100\ mph$ $(\simeq 5 \times 22)$

$$y_{max} \propto v_0^2 \qquad \simeq 25 \times 5\ m$$

$$\simeq 125\ \text{meters}$$

4 meters/story

~ 30 story bldg.

→ $t_{max}$ ?

$$v = v_0 - gt$$

$$0 = v_0 - g t_{max}$$

$$\boxed{t_{max} = \frac{v_0}{g} = \frac{10\ m/s}{10\ m/s^2} = 1\ s}$$

How long to fall back?
another 1s! $v$? $-v_0$.