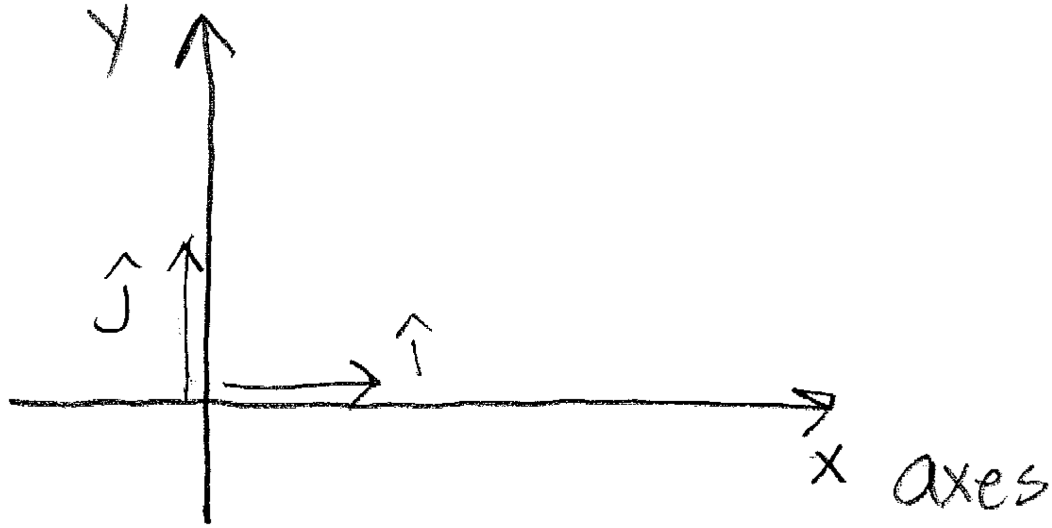
In 2-dimensions, we extend this concept to the $\perp$ direction:



CAUTION + OPPORTUNITY:

In any particular situation, one can choose x direction... need not be horizontal! Can also choose where origin is! Freedom = opportunity + danger

$\hat{i} \Rightarrow$ along +x direction

$\hat{j} \Rightarrow$ along +y direction ($\hat{y}$ too)

$|\hat{i}| = |\hat{j}| = 1$ no dimensions

$$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}| \cos \underset{\pi/2}{\theta} = 0 = \hat{j} \cdot \hat{i}$$

$$\underbrace{\hat{\imath} \times \hat{\jmath}}_{\text{direction, right hand rule}} = |\hat{\imath}||\hat{\jmath}| \sin \underset{\pi/2}{\theta} = +1$$

direction, right hand rule
out of page, +z

The base vector in the z direction is called $\hat{k}$

$\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\qquad$ $\hat{\jmath} \times \hat{\imath} = -\hat{k}$

"mnemonic" $\underbrace{1 \quad 2 \quad 3}_{\text{"normal order"}}$ $\qquad$ $\underbrace{2 \quad 1 \quad 3}_{\text{"flipped order"}}$

also

3 1 2	3 2 1
2 3 1	1 3 2
+ sign	− sign
$\hat{k} \times \hat{\imath} = \hat{\jmath}$	$\hat{k} \times \hat{\jmath} = -\hat{\imath}$
$\hat{\jmath} \times \hat{k} = \hat{\imath}$	$\hat{\imath} \times \hat{k} = \hat{\jmath}$

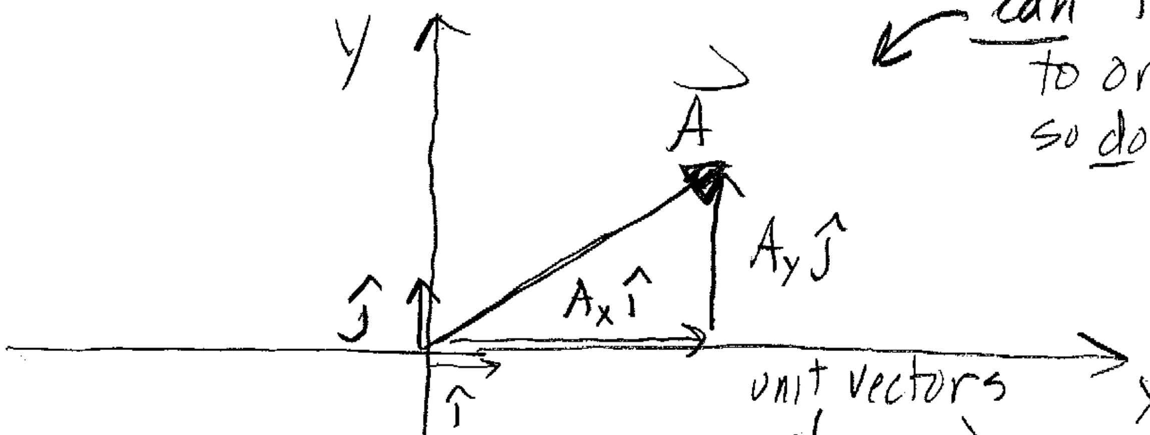
Vector Components

Geometric − • so far, this emphasized.
• conceptual
• independent of coordinate system

Algebraic − • depends on choice of coordinates

Algebraic – • practical

can translate to origin so do it



unit vectors

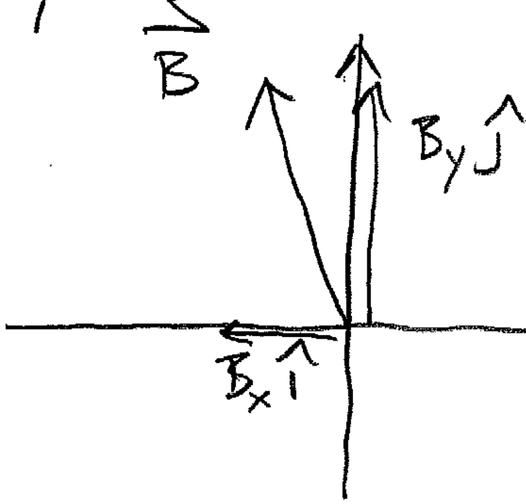
$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

might have dimensions like length, etc

components with dimensions

As drawn, components are both > 0

They can be < 0 !



$B_x < 0$

$B_y > 0$

Sometimes, we make a row n-tuple or row vector

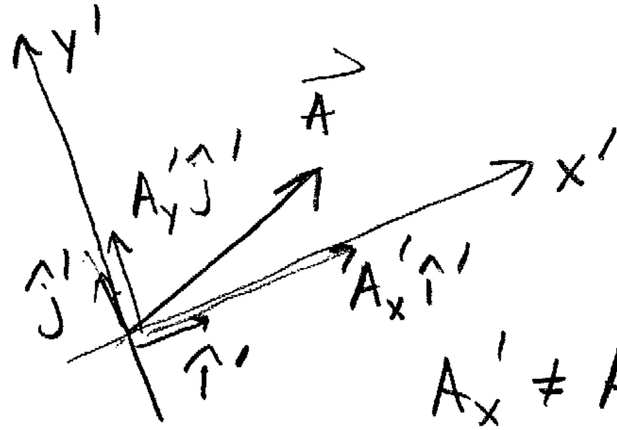
$\vec{A} = (A_x, A_y)$ $\qquad$ $\vec{B} = (B_x, B_y)$

or, if you are in 3-dim....

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

$\vec{A} = (A_x, A_y, A_z)$ etc.

Note, if a <u>rotated</u> coordinate system is used, a vector will have <u>different</u> coordinates ... + different base vectors



$A_x' \neq A_x$!

Components facillitate calculation:

<u>Addition</u>: $\vec{A} + \vec{B} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} + B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$

or

$\vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z)$

$= (A_x + B_x, A_y + B_y, A_z + B_z)$

<u>Example</u>: $\vec{A} = (3, 5, -7)$

p.9, 1.5 $\vec{B} = (2, 7, 1)$

$$\vec{A}+\vec{B} = (3+2, 5+7, -7+1)$$
$$= (5, 12, -6)$$

$$\vec{A}-\vec{B} = (3-2, 5-7, -7-1)$$
$$= (1, -2, -8)$$

Dot Product: (revert back to components

$$\vec{A}\cdot\vec{B} = \underbrace{(A_x\hat{\imath}+A_y\hat{\jmath}+A_z\hat{k})\cdot(B_x\hat{\imath}+B_y\hat{\jmath}+B_z\hat{k})}_{\text{9 terms but}}$$

$$\underbrace{\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{\imath}}_{\text{6 terms } 0} = 0 \quad \text{et. cetera}$$

3 remaining

$$= A_x B_x \underbrace{\hat{\imath}\cdot\hat{\imath}}_{1} + A_y B_y \underbrace{\hat{\jmath}\cdot\hat{\jmath}}_{1} + A_z B_z \underbrace{\hat{k}\cdot\hat{k}}_{1}$$

$$\boxed{\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z \quad \left(\begin{array}{c}\text{algebraic}\\ \text{dot product}\end{array}\right)}$$

Let's get the angle between $\vec{A}$ + $\vec{B}$ using the fact that

$$\boxed{\vec{A}\cdot\vec{B} = \underbrace{|\vec{A}|\,|\vec{B}|\cos\theta}_{\text{geometric dot product}}}$$

θ = angle between

First

$$|\vec{A}|^2 = \vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{3^2 + 5^2 + (-7)^2}$$

$$= \sqrt{83} \simeq 9.11$$

$$|\vec{B}| = \sqrt{2^2 + 7^2 + 1^2}$$

$$= \sqrt{54}$$

$$= 7.35$$

$$\vec{A}\cdot\vec{B} = 3\cdot 2 + 5\cdot 7 - 7\cdot 1 = 34$$

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

$$34 = 9.11 \;\; 7.35 \;\; \cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{34}{9.11\cdot 7.35}$$

$$\cos\theta = 0.507$$

$$\theta = 59.5^\circ = 1.04 \text{ radians}$$

Algebraic Cross Product

Stick with 2-d for $\vec{A}, \vec{B}$

$$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} \qquad \vec{B} = B_x \hat{\imath} + B_y \hat{\jmath}$$

$$\vec{A} \times \vec{B} = (A_x \hat{\imath} + A_y \hat{\jmath}) \times (B_x \hat{\imath} + B_y \hat{\jmath})$$

JUST ALGEBRA!

$$= A_x B_x \underbrace{(\hat{\imath} \times \hat{\imath})}_{=0} + A_x B_y (\hat{\imath} \times \hat{\jmath}) + (A_y B_x) \hat{\jmath} \times \hat{\imath} + A_y B_y \underbrace{(\hat{\jmath} \times \hat{\jmath})}_{=0}$$

$$\hat{\imath} \times \hat{\jmath} = +\hat{k} \qquad \hat{\jmath} \times \hat{\imath} = -\hat{k}$$

<u>2-d:</u>

$$\vec{A} \times \vec{B} = (A_x B_y - A_y B_x) \hat{k}$$ ← third dim pops out!

Suppose: $\vec{A} = (3,5) \qquad \vec{B} = (2,7)$

$$\vec{A} \times \vec{B} = (3 \cdot 7 - 5 \cdot 2) \hat{k}$$

$$= (21 - 10) \hat{k}$$

$$\boxed{\vec{A} \times \vec{B} = 11 \hat{k}}$$