Vectors

Quanties with both direction and magnitude.

Geometric Viewpoint:

like compass, straight-edge, "Side-Angle-Side"

Algebraic Viewpoint:

need coordinate axes... two different observers choose different axes, but knowing the geometry means results generally should not depend on that choice.

Tip: when you need axes, choose the one that makes the problem easiest!

Conceptually, # of dimensions is crucial!

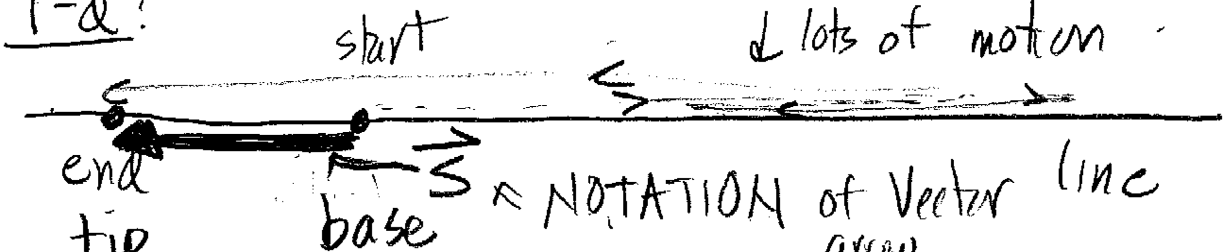
1 dimension ... # line, simple, illustrative
what origin?

2 dimensions: most problems in 20
what orientation?

3 dimensions: rarer in 20
begin to discuss

The first famous vector... <u>displacement</u>

<u>1-d</u>:



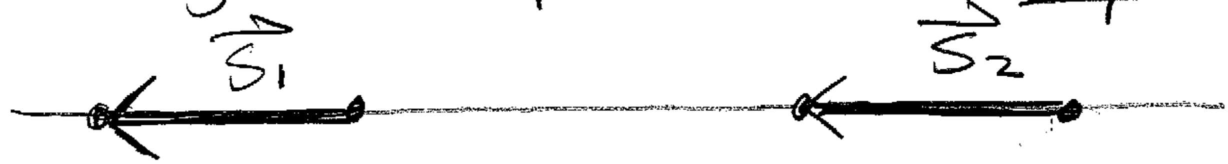
Imagine a "bug" begins, say, at $t=0$, at the "start" location, then wanders to the right, back to left, etc, finally ending at $t=T$ to the left, at, "end"!

The <u>direction</u> of the displacement vector is from the <u>start</u> to the <u>end</u>. All the intermediate wandering is <u>irrelevant</u> for the displacement. The vector displacement $\vec{s}$ is shown above.

Geometrically, that is pretty much the story, in 1-d!

Worth making this point: the

location of the beginning of a displacement vector is irrelevant. The following two displacements are <u>equal</u>:



| Length | $\equiv |\vec{S}_1|$ | Length | $\equiv |\vec{S}_2|$

To be equal, $|\vec{S}_1| = |\vec{S}_2|$ (magnitude)

right to left   right to left (direction)

Displacement is a "FREE" vector.

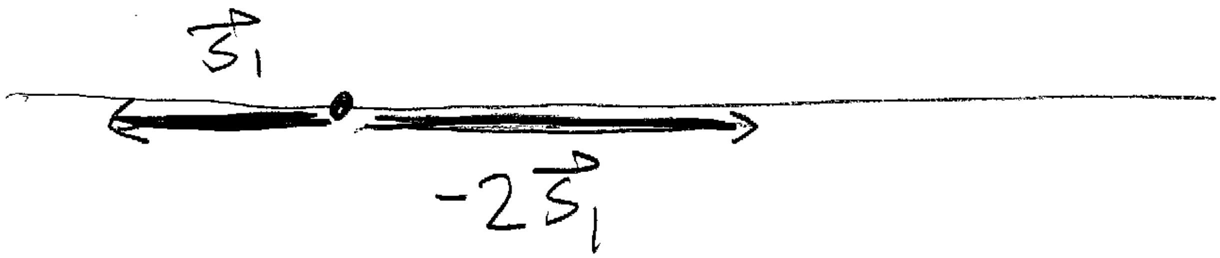
<u>opposite</u>: "BOUND" vector... like Forces when computing torques

Multiplication by a scalar $\begin{pmatrix}\text{1-d, same} \\ \text{for all-dim}\end{pmatrix}$

$b \Rightarrow$ real # $(+, 0, -)$

$b\vec{S}_1$ is: b times in magnitude

<u>flipped</u> <u>in</u> <u>direction</u> when $b < 0$



$b = -2$

Adding, multiplying, even dividing 1-d vectors... just like real #'s, better wait until 2 dimensions, 3 dimensions, etc.

Base Vector + Components

A base vector has no dimensions, only a direction. In one dimension, the base vector is usually called $\hat{\imath}$. (i-hat) (Sometimes called $\hat{x}$)

$$\vec{s}_1 = (\text{real number, units of length}) \cdot \hat{\imath}$$

<0 or >0 ?

usually points left to right

$$\vec{s}_1 = s_1 \hat{\imath}$$

base vector

the (in 1-d) component of $\vec{s}_1$!

To really specify s_1, need a system of units... METRIC SYSTEM! METERS!

Into the <u>Second Dimension</u> brings

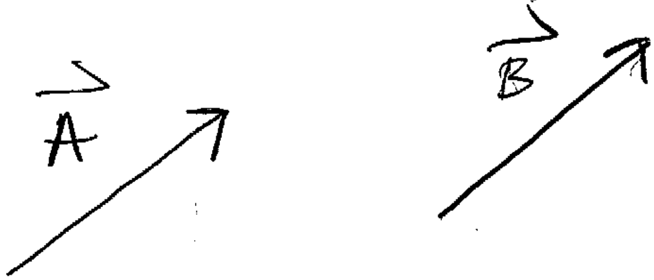
- addition of vectors that is more subtle than simply real #'s
- a type of multiplication that is also more subtle ... <u>scalar</u> or <u>dot</u> product. 1#

Into the <u>Third Dimension</u> brings

- two more types of multiplication
  1) <u>Vector</u> or <u>Cross</u> Product, which <u>DOES NOT COMMUTE</u>! 3#
  2) Tensor Product (not actually covered in Physics 20) 5#

Generalize... $\vec{A}$, $\vec{B}$, $\vec{C}$ vectors 2-d for now.

example



for a FREE vectors, (most common), as long as <u>lengths</u> same, directions same, $\vec{A} = \vec{B}$

$\vec{A}$'s unit vector is $\hat{A}$ (A-hat)

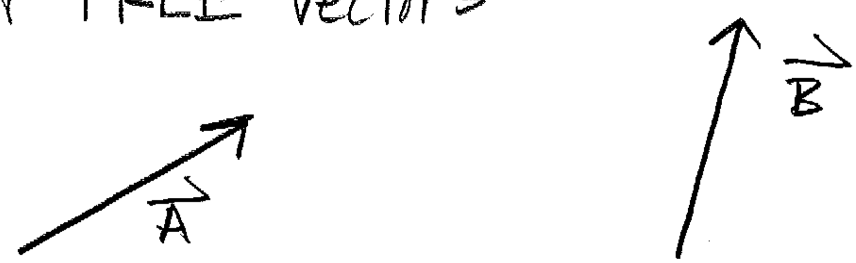
$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ no dimensions

"knows direction of $\vec{A}$"

$b = -\frac{1}{2}$, $b \cdot \vec{A} \Rightarrow$ draw that

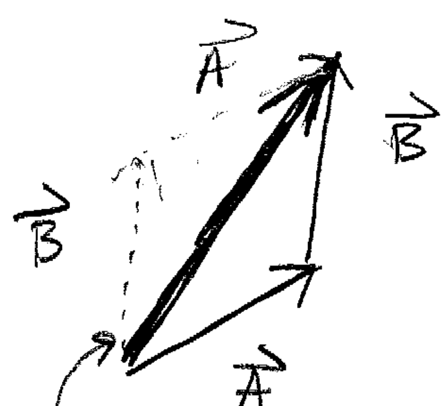
$b\vec{A}$ → direction flipped, half as long

Vector Addition in 2 or more dimensions for FREE vectors



"Geometric" addition... can translate (without rotation) $\vec{A}$ or $\vec{B}$.

$\vec{A} + \vec{B}$ ....



"tip to tail"

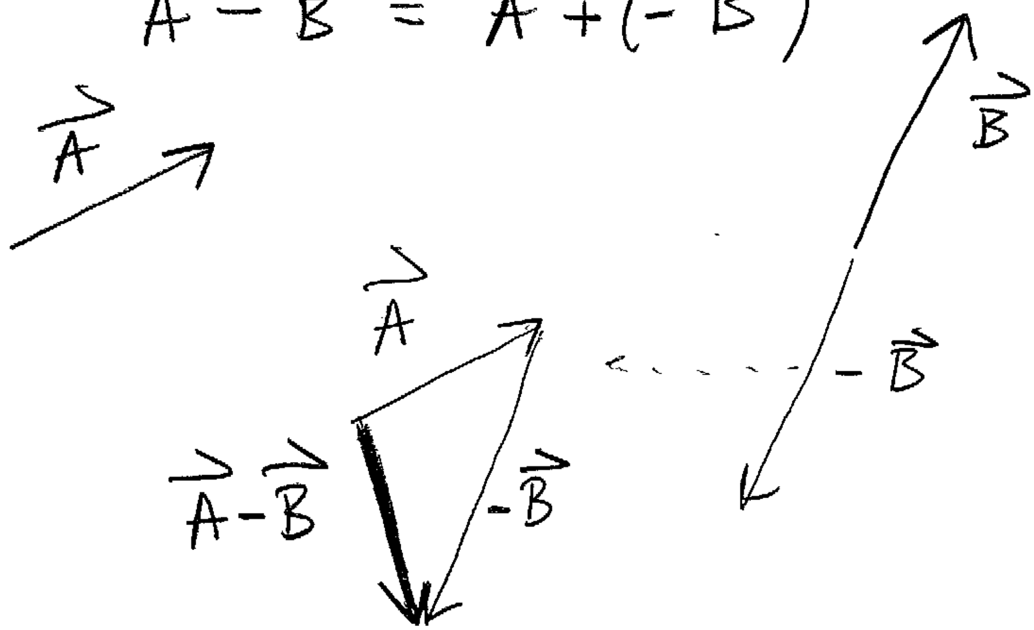
Note: $\vec{B} + \vec{A}$ gives same result!

$\vec{A} + \vec{B}$ "Resultant"

Vector <u>Addition</u> is <u>Commutative</u> (see the parallelogram on the previous page) $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Vector <u>Subtraction</u> is addition of the negative:

$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$



"<u>Triangle Inequality</u>" not in text

In $\boxed{\text{1 dimension}}$, adding vectors is like adding real numbers,

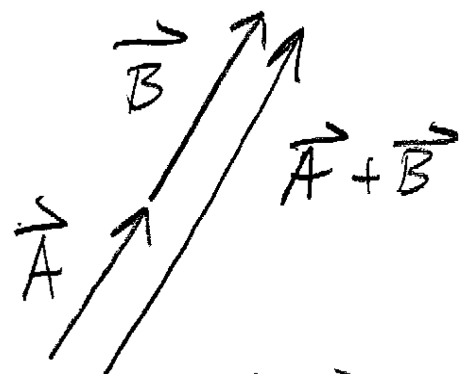
$$|\vec{A}+\vec{B}| = \begin{cases} |\vec{A}|+|\vec{B}| & \text{either} \quad (\text{relative sign } \vec{A} + \vec{B} \text{ same}) \\ \big||\vec{A}|-|\vec{B}|\big| & (\text{relative sign } \vec{A} + \vec{B} \text{ different}) \end{cases}$$

THINK: 1-dim, what conditions give $|\vec{A}-\vec{B}|=0$?

THINK: In 2 (or 3) dimensions, This Changes

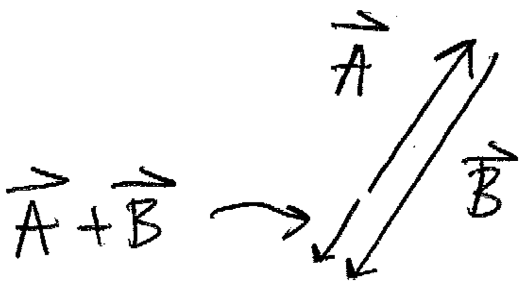
$$|\vec{A}+\vec{B}| \ \boxed{\text{only}} = |\vec{A}| + |\vec{B}|$$

when $\vec{A}$ and $\vec{B}$ are parallel (like 1-d)



$$|\vec{A}+\vec{B}| \ \boxed{\text{only}} = \big||\vec{A}| - |\vec{B}|\big|$$

when $\vec{A}$ and $\vec{B}$ are antiparallel



here, $|\vec{A}+\vec{B}| = \big||\vec{A}| - |\vec{B}|\big|$

In general, in 2 or more dimensions,

$$\big||\vec{A}|-|\vec{B}|\big| \leq |\vec{A}+\vec{B}| \leq |\vec{A}| + |\vec{B}|$$

"Triangle Inequality"