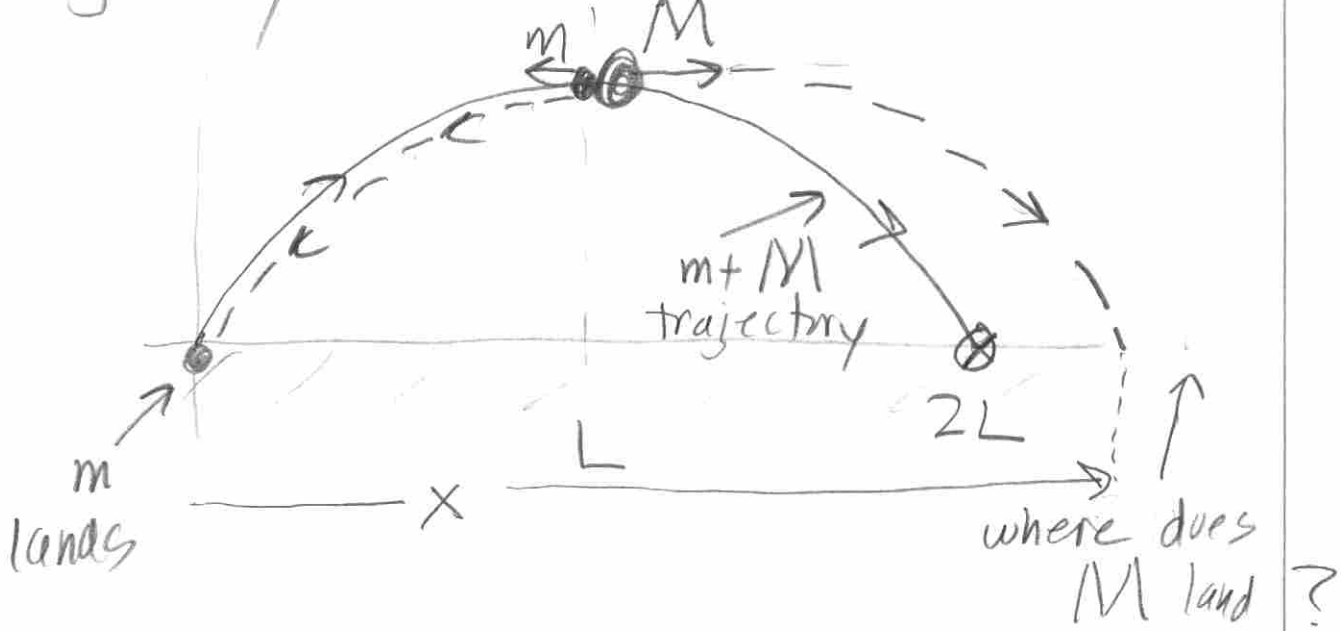
① KK 3.4

Center of mass follows standard trajectory and lands at 2L.



$$m \cdot 0 + Mx = 2L \cdot (m + M)$$

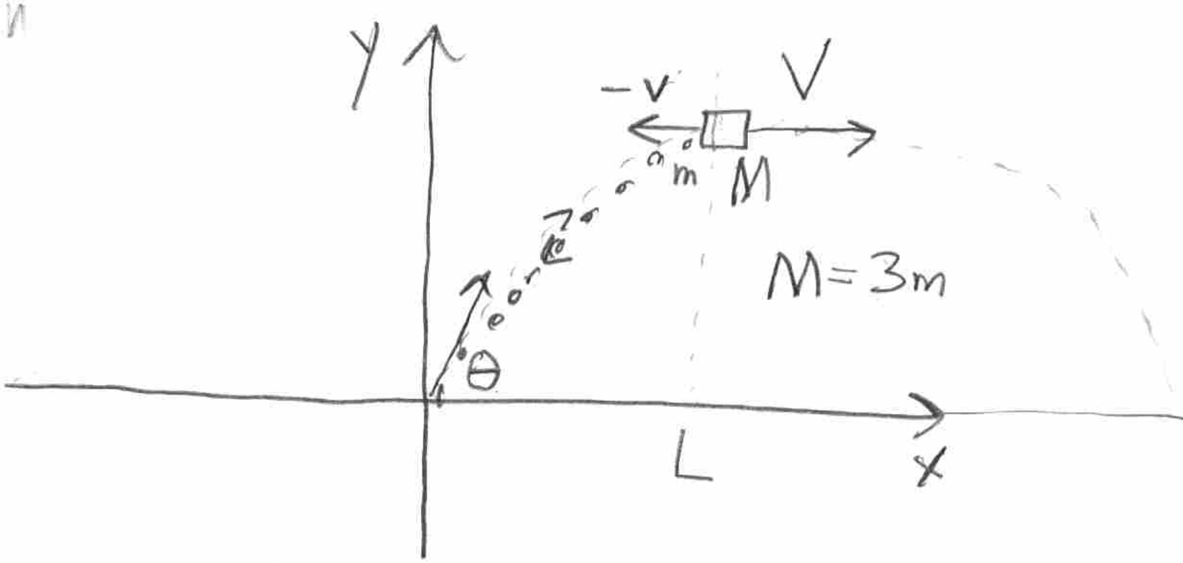
$$x = 2\left(\frac{m}{M} + 1\right) \cdot L$$

$$= 2 \cdot \left(\frac{1}{3} + 1\right) \cdot L$$

$$\boxed{x = \frac{8}{3} L}$$

(1) KK 3.4

again



$$V_{0x} = V_0 \cos\theta = V_x$$

time to get to L: $V_0 \cos\theta \, t = L$

$$t = \frac{L}{V_0 \cos\theta}$$

Whatever happens in the horizontal at the top of the trajectory, the time to fall is still t.

then at explosion, momentum conservation:

$$(-mv + MV) = (m+M)\, v_0 \cos\theta$$

to trace back to origin, $-v = -v_0 \cos\theta$

so $\quad V = \left(1 + 2\frac{m}{M}\right) v_0 \cos\theta$

and distance from origin to larger piece

$$= L + \left(1+2\frac{m}{M}\right) v_0 \cos\theta \cdot \frac{L}{v_0 \cos\theta} \quad \boxed{= 2\left(1+\frac{m}{M}\right)L}$$

$$= 2\left(1+\frac{m}{M}\right)L = \frac{8}{\ }$$

② KK 3.5

recall $v_y = v_{y0} - gt \Big] \rightarrow t = \dfrac{v_{y0} - v_y}{g}$

$$y = v_{y0}t - \frac{1}{2}gt^2$$

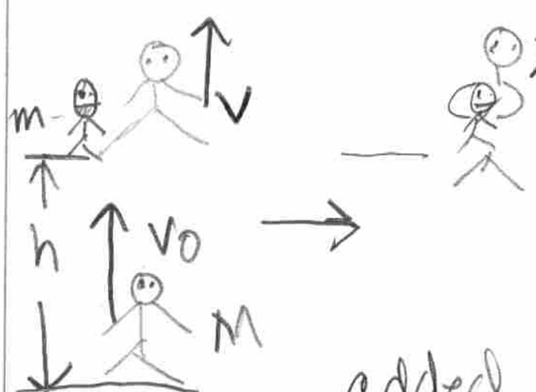
$$y = v_{y0}\left(\frac{v_{y0}-v_y}{g}\right) - \frac{1}{2}g\frac{(v_{y0}-v_y)^2}{g^2}$$

$$= \frac{(v_{y0}-v_y)}{g}\left[v_{y0} - \frac{1}{2}(v_{y0}-v_y)\right]$$

$$= \frac{1}{2}\frac{1}{g}(v_{y0}-v_y)(v_{y0}+v_y)$$

$$y = \frac{1}{2g}(v_{y0}^2 - v_y^2)$$

or $v_y^2 = v_{y0}^2 - 2gy$



$$v^2 = v_0^2 - 2gh$$

$$Mv + m\cdot 0 = (m+M)v'$$

$$v' = \frac{M}{m+M}v = \frac{M}{m+M}\sqrt{v_0^2 - 2gh}$$

<u>added</u> height h' satisfies:

$$0 = v'^2 - 2gh'$$

$$h' = \frac{1}{2g}v'^2 = \frac{1}{2g}\left(\frac{M}{m+M}\right)^2(v_0^2 - 2gh)$$

so $\boxed{h + h' = \left(\frac{M}{m+M}\right)^2\frac{v_0^2}{2g} + \left(1 - \left(\frac{M}{m+M}\right)^2\right)h}$ a mean of $\frac{v_0^2}{2g}$ + h.

(3) KK 3.8

$$v_y = v_{y0} - gt \Rightarrow t = (v_{y0} - v_y)/g$$

$$y = v_{y0}t - \frac{1}{2}gt^2$$

$$= v_{y0}\left(\frac{v_{y0}-v_y}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}-v_y}{g}\right)^2$$

$$= \frac{(v_{y0}-v_y)}{g}\left[v_{y0} - \frac{1}{2}(v_{y0}-v_y)\right]$$

$$y = \frac{1}{2g}(v_{y0}-v_y)(v_{y0}+v_y) = \frac{1}{2g}(v_{y0}^2 - v_y^2)$$

$$v_{y0}^2 = v_y^2 + 2gy \quad \text{at top, } v_y = 0, \; y = h$$

$$v_{y0}^2 = 2gh$$

$$v_{y0} = \sqrt{2gh}$$

Impulse:

$$\int_0^t \vec{F}\,dt = \vec{I} = \vec{P}(t) - \vec{P}(0)$$

$m v_{y0} \hat{j}$

$\vec{v}_0 = v_{y0}\hat{j}$

$\vec{v} = 0$

$$\vec{F} = \vec{F}_{ground} - \vec{W}$$

$$= \vec{F}_{ground} - mg\hat{j}$$

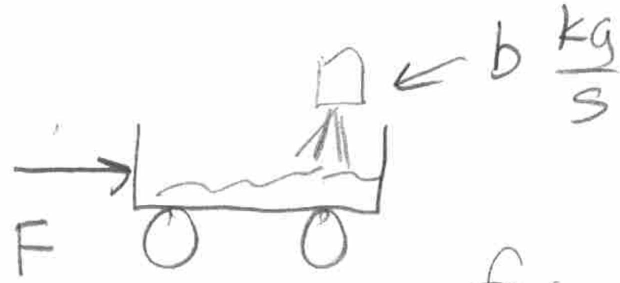
$$\vec{I} = \int_0^t \vec{F}_{ground}\,dt - mg\hat{j}\,\Delta t = m\sqrt{2gh}\,\hat{j}$$

$$\int_0^t \vec{F}_{ground}\,dt = mg\hat{j}\,\Delta t + m\sqrt{2gh}\,\hat{j}$$

$$= 50 \cdot 9.8 \cdot \frac{1}{5} + 50\sqrt{2 \cdot 9.8 \cdot 0.8}$$

$$\boxed{\int_0^t \vec{F}_{ground}\,dt = (98 + 198)\hat{j} = 296\hat{j}\ \frac{kg \cdot m}{s}}$$

(4) KK 3.10



railcar mass M

in time t, constant force F provides impulse Ft. This equals the change in momentum of the system, which equals $(M+m)v$ $\quad m = bt$

$$Ft = (M+bt)v$$

$$\boxed{v = \frac{Ft}{M+bt}}$$

$F = 100\,N$, $t = 10\,s$, $M = 500\,kg$
$b = 20\,kg/s$,

$$\boxed{v = \frac{100 \cdot 10}{500 + 20 \cdot 10} = \frac{1000}{700} \cong 1.4\,m/s}$$

(5) KK 3.16

In time t, mass $\rho \cdot V$ $\quad \rho =$ mass density, $V =$ volume released, $V = \pi \left(D/2\right)^2 \cdot V_0 t$. Momentum transferred ... $\rho V V_0 = \frac{\pi}{4} \rho D^2 V_0^2 t$ = impulse = Ft

$$\boxed{F = \frac{\pi}{4} \rho D^2 V_0^2}$$