Physics 20 Problem Set 7

Harry Nelson

due Monday, November 15, by 5pm
to the Physics 20 Boxes in Broida Hall's Lobby

Course Announcments: These problems pertain to the sixth week's lectures, and the corresponding reading is pages 80-86 and 97-101 of KK, Chapter 16, pages 343-352 and Chapter 15, pages 317-320 of RHK4. Also, PSR fellows are available to help you on Wed. and Thurs., 6pm-8pm in the PSR, and Sunday 6pm-8pm in 1640 Broida. Take advantage of their help!!!

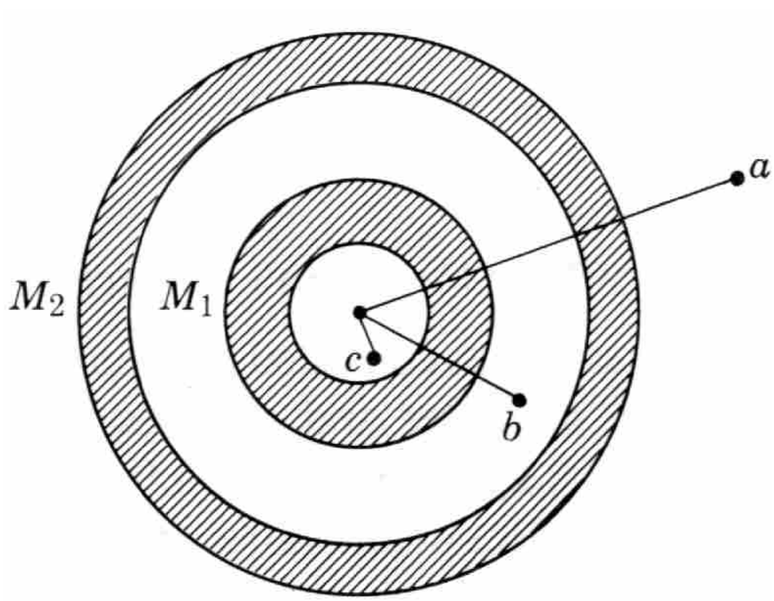

Figure 1: Problem 1.

1. Two concentric shells of uniform density having masses M_1 and M_2 are situated as shown in Fig. 1. Find the force on a particle of mass m when the particle is located at:

   (a) $r = a$
   (b) $r = b$
   (c) $r = c$

   The distance r is measured from the center of the shells. (RHK4 16.20)

2. For this problem, assume that the density of the Earth is uniform. Show that, at the bottom of a vertical mine shaft dug to depth D, the measured value of g will be:

$$g = g_s \left(1 - \frac{D}{R_E}\right)$$

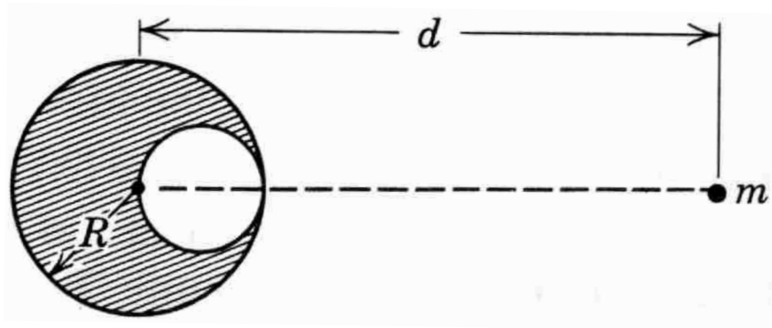

Figure 2: Problem 3.

where g_s is the acceleration caused by gravity at the surface, and R_E is the radius of the Earth. (RHK4 16.22)

3. A sphere with radius R and otherwise uniform mass density has a hollow, as shown in Fig. 2, where the hollow is also spherical with radius $R/2$, and just touches the outside surface of the massive sphere and also passes through the massive sphere's center. With what force will the hollowed sphere attract a small sphere of mass m, which is a distance d from the center? The mass of the sphere before it was hollowed was M.

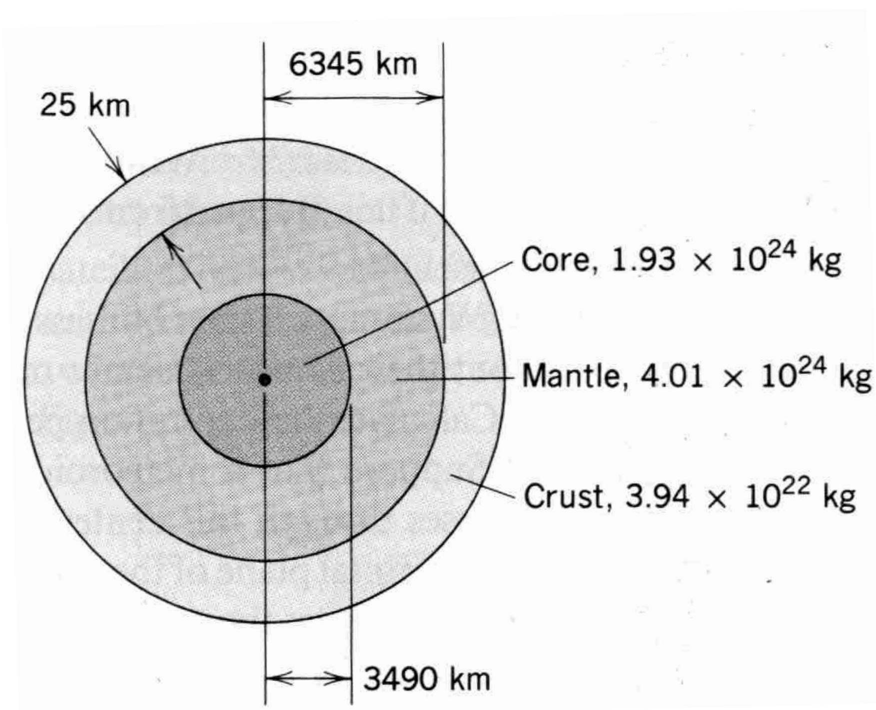

Figure 3: Problems 4 and 5. Assume that within the zones, the mass density is uniform.

4. Fig. 3 shows, not to scale, a cross section through the interior of the Earth. Rather than being uniform throughout, the Earth is divided into three zones: an outer *crust*, a *mantle*, and an inner *core*. The dimensions of these zones and the mass contained within them are shown in Fig. 3. The Earth has a total mass 5.98×10^{24} kg and radius 6370 km. Ignore rotation and assume that the Earth is spherical. Do both symbolic and numerical answers to 3 significant digits. (RHK4 16.25)

(a) Calculate g at Earth's surface.

(b) Suppose that a shaft is excavated to the border of the crust and the mantle; what value of g will be measured at the bottom?

(c) Now repeat the calculation of g at the depth of the crust/mantle interface under the assumption that the density of the Earth is uniform. Use the result of Problem 2.

5. Use the model of the Earth shown in Fig. 3 to study the variation of g with depth in the interior of the Earth. There are surprises, such as that in Problem 4.

(a) The important differences between a sphere of uniform mass density and the more realistic Earth model in Fig. 3 are the boundaries between regions of *different* densities. Compile a table with three columns: make column 1 the numerical radii of the boundaries, which we'll call $\tilde{r}$; column 2, the numerical *average mass density* for the Earth at radii smaller than the boundary, which we'll call $\rho_<$, in units of $10^3\,\mathrm{kg/m^3}$; and column 3, the numerical *local mass density* for the Earth at radii larger than the boundary which we'll call $\rho_>$, but don't consider radii so large as to cross the next boundary, in the same units. To calculate the average mass densities, you may have to combine some of the information in Fig. 3. There are three rows in the table, corresponding to the core/mantle, mantle/crust, and crust/air boundaries; assume the density of air is 0.

(b) Imagine the value of g outside a density boundary, which is at radius $\tilde{r}$. Call the mass density at radii smaller than $\tilde{r}$ $\rho_<$, and the mass density at radii greater than $\tilde{r}$ $\rho_>$. Show that the value of g for radii $r > \tilde{r}$, but less than the radius of the next boundary, has the form

$$g = G\left(\frac{A}{r^2} + Br\right). \tag{1}$$

Evaluate A and B symbolically in terms of $\rho_<$, $\rho_>$, $\tilde{r}$, and fundamental constants. Without any algebra and by just thinking about Problem 2, you should be able to reason out the value of A for $\rho_< = \rho_>$.

(c) Find g at the core-mantle interface, symbolically and numerically, and give the very simple symbolic and numerical equation for g as a function of distance r from the center of the Earth to the core-mantle interface. You can do this without using the results of the previous two parts; the information Fig. 3 is sufficient.

(d) Show that g has a local *minimum* within the mantle. It is easiest to do this symbolically with Equation 1, leaving A and B in the formula for r_{min}. However, using the detailed expressions for A and B in terms of other quantities is necessary to make sure the radius r_{min} which minimizes g is actually in the mantle and not in the core or crust. Find the numerical distance from the Earth's center where the minimum occurs and the associated symbolic and numerical value of g_{min}.

(e) Make a numerical graph showing g as a function of radius from the center of the Earth through the surface of the Earth to twice the radius of the surface. (RHK4 16.26)

6. KK 2.26

7. KK 2.31