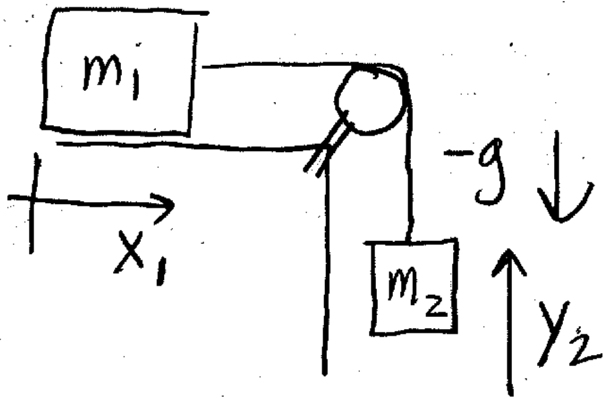
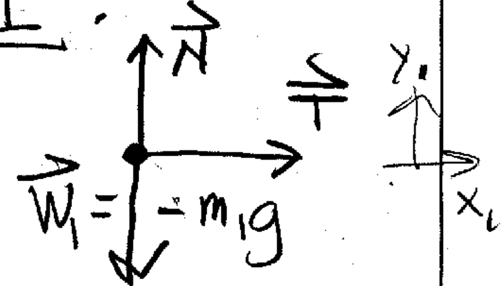
① KK 2.2



Block 1:



• no acceleration in y direction ...

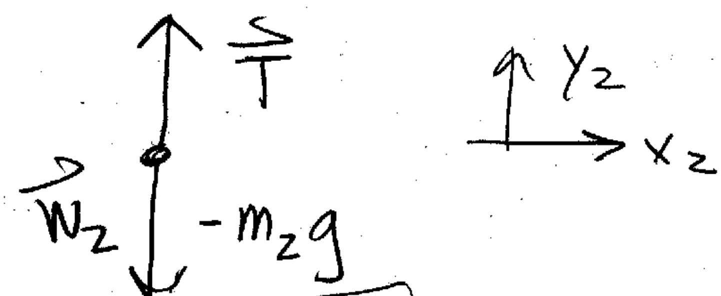
• I $\quad \boxed{T = m_1 \ddot{x}_1}$

net force $\qquad$ (mass)·(acceleration)

$\Rightarrow$ introduced 2 unknowns!

$T, \ddot{x}_1$

Block 2:



II $\quad \boxed{T - m_2 g = m_2 \ddot{y}_2}$

net force $\qquad$ (mass)·(acceleration)

$\Rightarrow$ introduced 3rd unknown

$\ddot{y}_2$

Constraint: $\quad \boxed{\ddot{x}_1 = -\ddot{y}_2}$ ← length of string constant

III

want $\ddot{x}_1$, eliminate $\ddot{y}_2$ first

(plug III into II)

$$T - m_2 g = m_2(-\ddot{x}_1)$$

now eliminate T with equation I:

$$m_1\ddot{x}_1 - m_2 g = -m_2\ddot{x}_1$$

$$(m_1 + m_2)\ddot{x}_1 = m_2 g$$

$$\boxed{\ddot{x}_1 = \left(\frac{m_2}{m_1 + m_2}\right) g}$$

Choose: $x_{01} = 0$

$v_{01} = 0$ since blocks released from rest

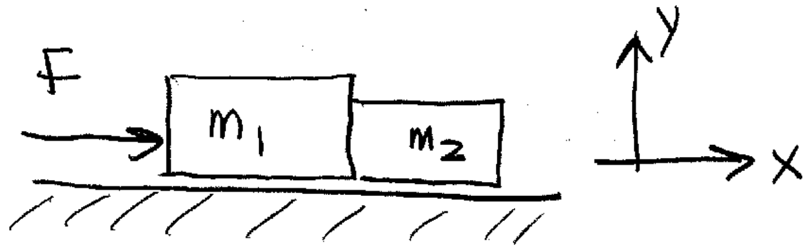
$$\therefore \boxed{x_1 = \frac{1}{2}\left(\frac{m_2}{m_1+m_2}\right) g t^2}$$

<u>clue</u>: $m_1 = m_2$

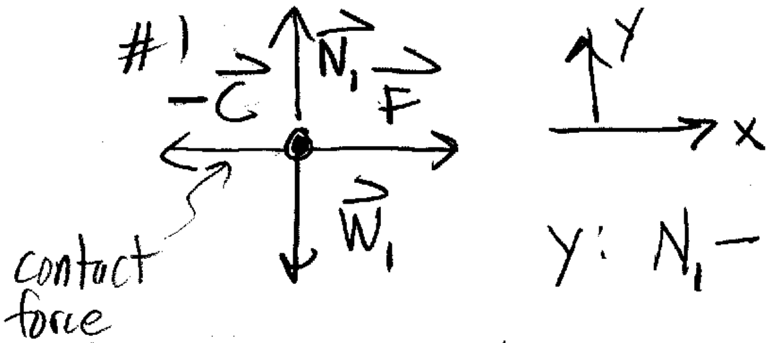
$$x_1 = \frac{1}{2}\left(\frac{m_2}{m_2+m_2}\right) g t^2$$

$$x_1 = \frac{1}{4} g t^2 \checkmark$$

(2) KK 2.3



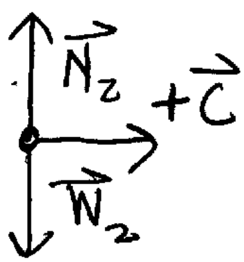
#1



$y: \; N_1 - m_1 g = 0$ } not pertinent

$x: \; F - C = m_1 \ddot{x}_1$ (I)

#2



$y: \; N_2 - m_2 g = 0$ } not pertinent

$C = m_2 \ddot{x}_2$ (II)

Constraint: $\ddot{x}_1 = \ddot{x}_2$ (III)

Givens: F, m_1, m_2

Unknowns: (horizontal) $C, \ddot{x}_1, \ddot{x}_2$ (3)

3 equations ... GOOD

Want: C ... eliminate $\ddot{x}_1$ + $\ddot{x}_2$

$C = \underset{\text{(II)}}{m_2 \ddot{x}_2} = \underset{\text{(III)}}{m_2 \ddot{x}_1}, \quad \ddot{x}_1 = \dfrac{C}{m_2}$ plug into (I)

$$F - C = m_1 \cdot \ddot{x}_1 = m_1 \cdot \frac{C}{m_2}$$

$$F = \left(1 + \frac{m_1}{m_2}\right) C$$

$$\boxed{C = \frac{F}{1 + \frac{m_1}{m_2}}} = \frac{3\,N}{1 + \frac{2\,kg}{1\,kg}} = \frac{3\,N}{3}$$

symbolic

$$= 1\,N$$

numerical

$F = 3N$
$m_1 = 2\,kg$
$m_2 = 1\,kg$

<u>more checks:</u>

$$F - C = 3 - 1 = 2\,N = m_1 \ddot{x}_1$$

$$2N = 2\,kg \cdot \ddot{x}_1$$

$$\boxed{\ddot{x}_1 = \frac{2N}{2\,kg} = 1\,m/s^2}$$

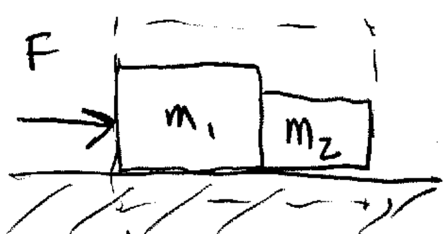
$$C = m_2 \ddot{x}_2$$

$$1N = 1\,kg \cdot \ddot{x}_2$$

$$\boxed{\ddot{x}_2 = 1\,m/s^2 = \ddot{x}_1}$$

checks

Also ... another way



$$\underbrace{(m_1 + m_2)}_{} = 3\,kg$$

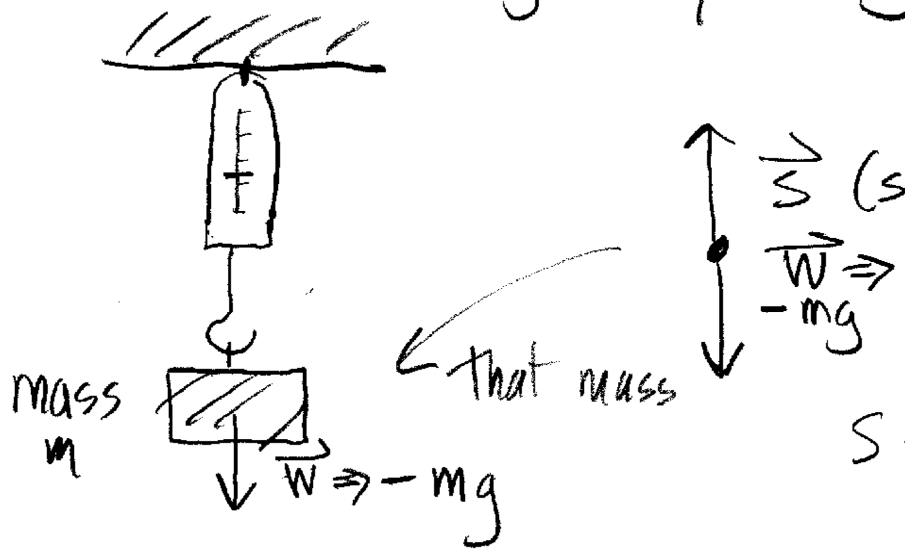
$$F = (m_1 + m_2)\ddot{x}$$

$$\ddot{x} = \frac{F}{m_1 + m_2} = \frac{3N}{3\,kg} = 1\,m/s^2$$

checks ✓

$$\boxed{C = m_2 \ddot{x} = \frac{m_2}{m_1 + m_2} F = \frac{1}{1 + \frac{m_1}{m_2}} F}$$ ✓

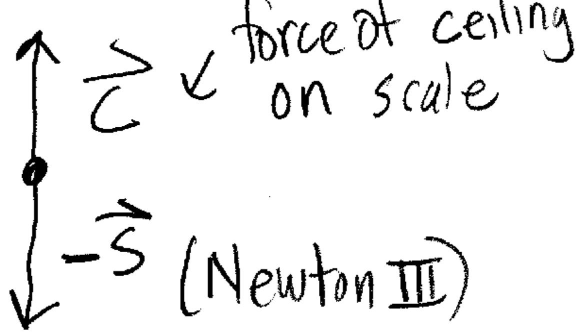
3a) <u>Key point</u>: scales that <u>are at rest</u> measure weight on them, and always transmit the weight hung on them to the ceiling they hung on!



$S - mg = 0$ (no accel).

$S = mg$

Remember, <u>weight</u> of mass m <u>only</u> applies to <u>it</u>! The scale applies force $\vec{S}$ to the mass, which, when added to $-mg$, gives no net force, to keep the mass at rest. OK, so now draw diagram for the <u>scale</u>:

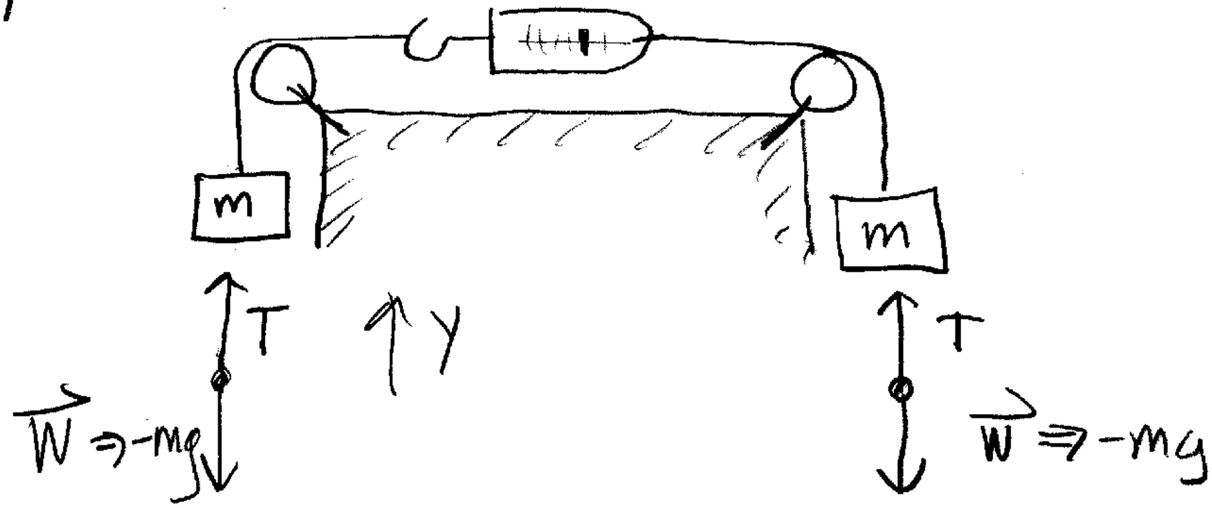


scale at rest

$C - S = 0$

$C = S = mg$

(a) Now, turn the scale to the right, and the ceiling is replaced by the 10 lb mass on right; the original mass by the 10 lb mass on the left



at rest

$$T - mg = 0$$
$$T = mg$$

$$T - mg = 0$$
$$T = mg$$

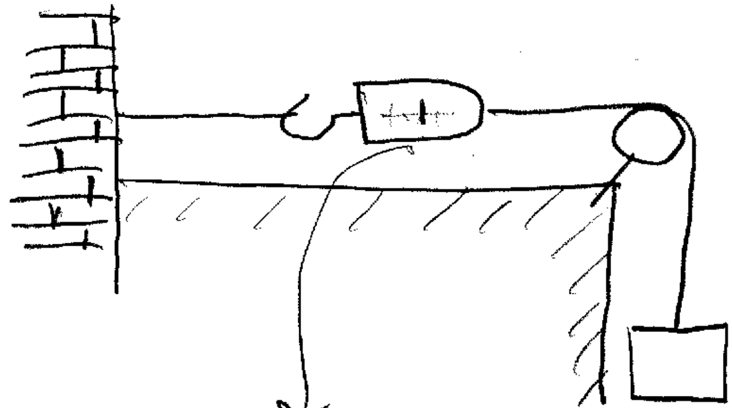
The tension T is transmitted by the cord to both ends of the scale, just like $-\vec{S}$ and $\vec{C}$ previously.

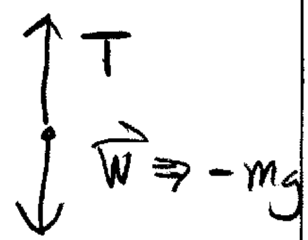
So, scale now reads $\boxed{T = mg = 10\text{ lbs}}$

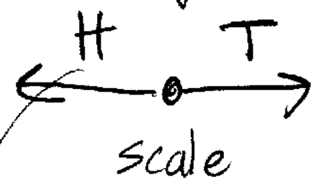
Remember, pounds are the english units for mg

(type "10 pounds in newtons" in google ... 44.5 N)

(b)







$T - mg = 0$

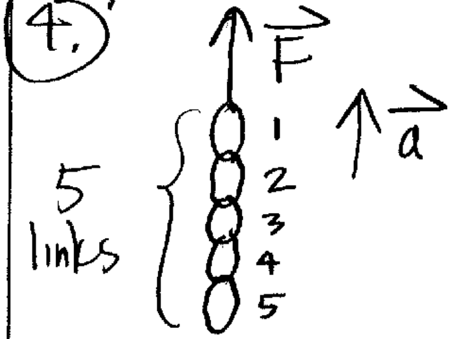
$T = mg$

Hook $T - H = 0$ (scale at rest)

$H = T$ (force on hook is T)

$\boxed{\text{So, again, scale reads } T = mg = 10 \text{ lbs}}$

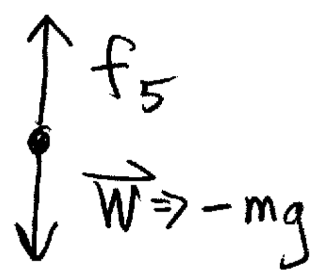
4.



each link $m = 0.1\,kg$

$a = 2.5\ m/s^2$

Most clever: look at link #5 first.

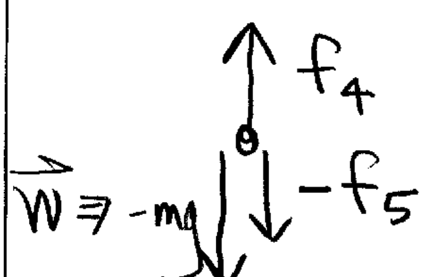


$$\underbrace{f_5 - mg}_{\text{net force}} = \underset{\text{acceleration}}{ma}$$

f_5: force between links 4 & 5

ⓐ $$\boxed{\begin{aligned} f_5 &= m(g+a) \\ &= 0.1(9.8+2.5)\ N \\ &= 1.23\ N \end{aligned}}$$

NOW LOOK AT link #4.



$-f_5$: from Newton's 3rd

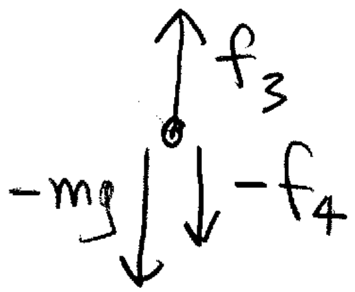
$$\underbrace{f_4 - f_5 - mg}_{\text{net force}} = ma$$

f_4: force between links 3 & 4

$$f_4 = f_5 + m(g+a)$$

ⓐ $$\boxed{\begin{aligned} f_4 &= 2m(g+a) \\ &= 2.46\ N \end{aligned}}$$

NOW LOOK AT link # 3



$-f_4$: from Newton's 3rd

$$f_3 - f_4 - mg = ma$$

f_3: force between links 2+3

(a)
$$\boxed{\begin{aligned} f_3 &= f_4 + m(g+a) \\ &= 3m(g+a) \\ &= 3.69\ \text{N} \end{aligned}}$$

GET THE HANG OF IT ?

(a) $\boxed{f_2 : \; = 4m(g+a) = 4.92\ \text{N}}$

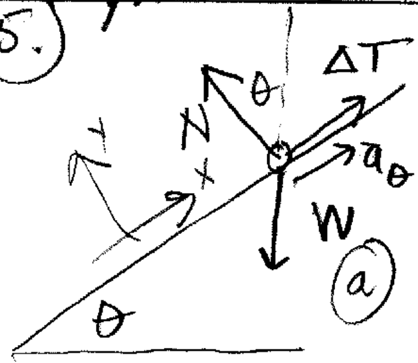
(b) $\boxed{F = f_1 = 5m(g+a) = 6.15\ \text{N}}$

(c) Net Force = ma EVERY TIME

$$= 0.1 \cdot 2.5$$
$$= 0.25\ \text{N}$$

<u>NOTE</u>: • Worst place to catch your finger: between links 1+2
• Link #1 would break first!

5.



use x-y coordinates with x ∥ to cables.

(a) x: $\Delta T - W\sin\theta = ma_\theta$

$$\boxed{\Delta T = ma_\theta + W\sin\theta}$$

$m = 2800\text{ kg}$ $\quad a_\theta = 0.81\text{ m/s}^2$ $\quad W = mg$

$g = 9.8\text{ m/s}^2$

$\theta = 35° = 0.611\text{ radians}$

$\Delta T = 2800 \cdot 0.81 + 2800 \cdot 9.8 \cdot \sin(0.611)$

$$\boxed{\Delta T = 1.80 \cdot 10^4\text{ N}}$$

(b) y: $N - W\cos\theta = 0$

$$\boxed{N = W\cos\theta}$$

$= 9.8 \cdot 2800 \cdot \cos(0.611)$

$$\boxed{N = 2.25 \cdot 10^4\text{ N}}$$

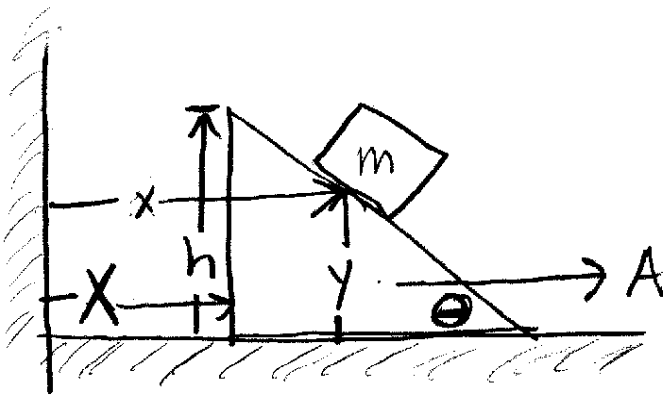
note: if $a_\theta = 0$, $\sqrt{\Delta T^2 + N^2} = W$

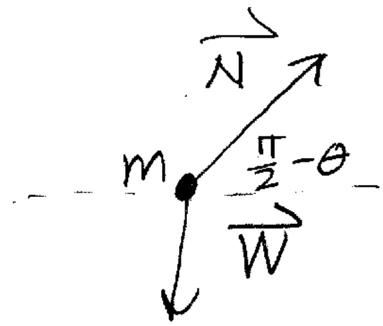
$= 27{,}440\text{ N}$

for $a_\theta = 0.81\text{ m/s}^2$,

$\sqrt{\Delta T^2 + N^2} = 28{,}801\text{ N}$.

6





$\theta = 45^\circ = \pi/4$

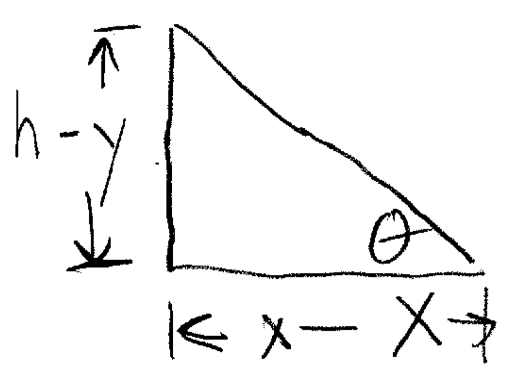
Newton's Second Law on Block

$$x: \quad m\ddot{x} = N_x = N\cos\left(\tfrac{\pi}{2}-\theta\right) = N\sin\theta$$

$$y: \quad m\ddot{y} = N_y - W = N\sin\left(\tfrac{\pi}{2}-\theta\right) - mg$$
$$= N\cos\theta - mg$$

3 unknowns: $\ddot{x}$, $\ddot{y}$, and N. But only 2 equations!

3rd equation: constraint



$$\underbrace{\tan\theta}_{\text{constant}} = \frac{h-y}{x-X}$$

$$(x - \underset{\substack{\uparrow \\ \text{given, } \ddot{X} = A}}{X})\tan\theta = \underset{\substack{\uparrow \\ \text{constant}}}{h} - y$$

take 2 derivatives: $(\ddot{x} - \ddot{X})\tan\theta = -\ddot{y}$

or, $\ddot{y} = (A - \ddot{x})\tan\theta$

so $m(A-\ddot{x})\tan\theta = N\cos\theta - mg$

$$m\ddot{x} = N\sin\theta$$

$$\text{or} \quad N = \frac{m\ddot{x}}{\sin\theta}$$

and $\cancel{m}(A-\ddot{x})\tan\theta = \dfrac{\cancel{m}\ddot{x}\cos\theta}{\sin\theta} - \cancel{m}g$

$$A\tan\theta + g = \left(\frac{1}{\tan\theta} + \tan\theta\right)\ddot{x}$$

$$\frac{1}{\tan\theta} + \tan\theta = \frac{1+\tan^2\theta}{\tan\theta} = \frac{\dfrac{\cancel{\cos^2\theta + \sin^2\theta}^{\,1}}{\cos^2\theta\ \cos\theta}}{\dfrac{\sin\theta}{\cancel{\cos\theta}\ 1}}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

so $A\tan\theta + g = \dfrac{\ddot{x}}{\sin\theta\cos\theta}$

$$\ddot{x} = \sin\theta\cos\theta(A\tan\theta + g)$$

$$\boxed{\ddot{x} = A\sin^2\theta + g\sin\theta\cos\theta}$$

$$\ddot{y} = (A - A\sin^2\theta - g\sin\theta\cos\theta)\tan\theta$$

$$= (A(1-\sin^2\theta) - g\sin\theta\cos\theta)\tan\theta$$

$$= \left(A\cos^2\theta\cdot\frac{\sin\theta}{\cos\theta} - g\sin\theta\cos\theta\frac{\sin\theta}{\cos\theta}\right)$$

$$\boxed{\ddot{y} = A\sin\theta\cos\theta - g\sin^2\theta}$$

$\theta = \pi/4 = 45^\circ$

$$\ddot{x} = A \cdot \underbrace{\sin^2 \pi/4}_{(1/\sqrt{2})^2} + g \underbrace{\sin \pi/4 \cos \pi/4}_{(1/\sqrt{2})(1/\sqrt{2})}$$

$$\boxed{\ddot{x} = \frac{1}{2}(A+g)}$$

$$\ddot{y} = A \sin \pi/4 \cos \pi/4 - g \sin^2 \pi/4$$

$$\boxed{\ddot{y} = \frac{1}{2}(A-g)} \rightarrow \text{hint, } A = 3g$$

$$\ddot{y} = \frac{1}{2}(3g-g) = g$$