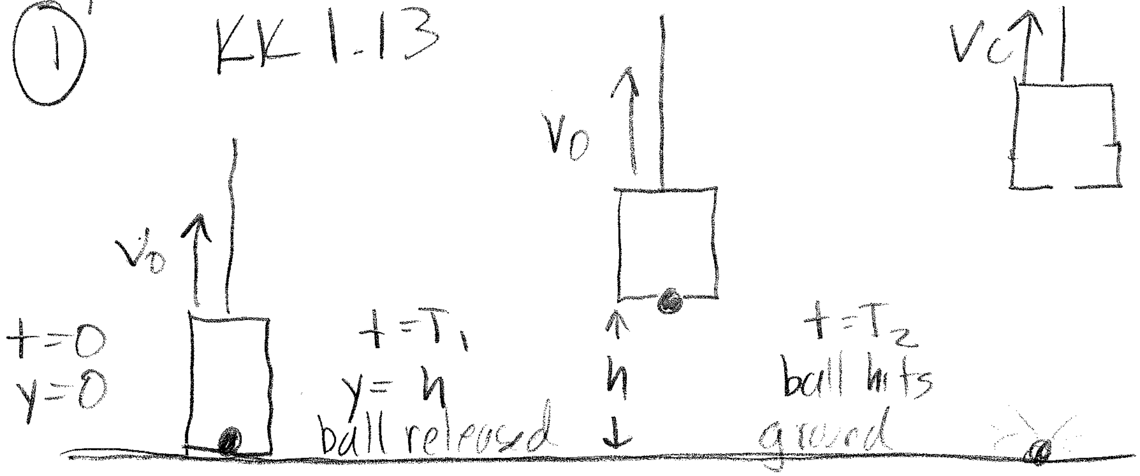
① KK 1.13



$$h = v_0 T_1$$

ball $y(t) = h + v_0 t - \frac{1}{2} g t^2 = 0$ (ball hits ground)

given: T_1 & T_2

solve for: v_0 & then $h = v_0 T_1$

$$v_0(T_1 + T_2) - \frac{1}{2} g T_2^2 = 0$$

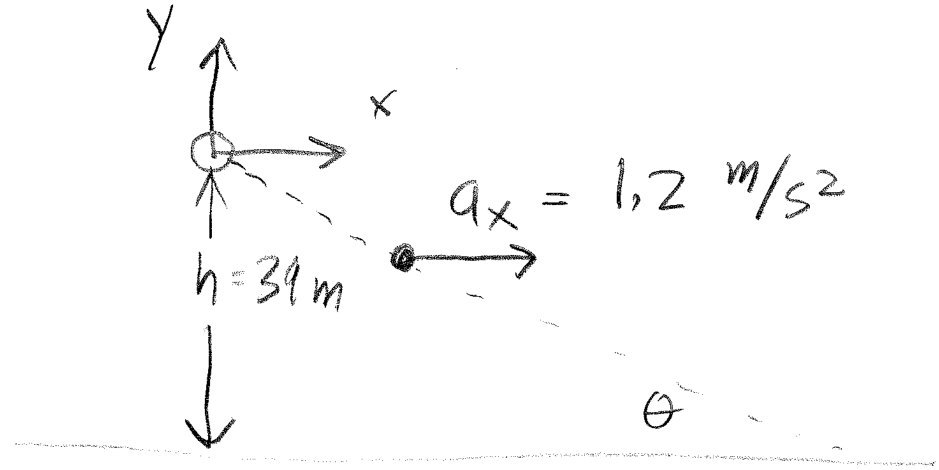
$$v_0 = \frac{\frac{1}{2} g T_2^2}{T_1 + T_2}$$

$$\boxed{\text{so } h = v_0 T_1 = \frac{g T_2^2 T_1}{2(T_1 + T_2)}}$$

eg $T_1 = T_2 = 4\,s$,

$$h = \frac{9.8 \cdot \cancel{4}^{2} \cdot \cancel{4}^{1}}{\cancel{2} \cdot \cancel{(4+4)}_{\cancel{8}\,\cancel{4}}} = 39.2\,m$$

(2) Put the origin of the coordinates on the ball.



$$y = -\frac{1}{2} g t^2 \qquad x = \frac{1}{2} a_x t^2$$

(a) so, $\frac{y}{x} = \frac{-\frac{1}{2} g t^2}{\frac{1}{2} a_x t^2} = -\frac{g}{a_x} = \underline{\underline{\text{constant}}}$

Therefore motion is a straight line.

That is, $y = -\frac{g}{a_x} x$

and, $\tan\theta = \frac{|y|}{x} = \frac{g}{a_x}$

$$\tan\theta = \frac{9.8}{1.2} = 8.16\overline{6}$$

$$\boxed{\theta = 1.45 \text{ radians} = 83.0°}$$

$$\tan\theta = \frac{h}{R}$$

$$\boxed{R = \frac{h}{\tan\theta} = \frac{h}{g/a_x} = \frac{39\,m}{(9.8/1.2)} = 4.78\,m}$$

(b) $\frac{1}{2}gt^2 = h$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 39{,}0\ m}{9{,}8\ m/s^2}} = 2{,}82\ s$$

(c) $v_y = -gt$ $\qquad$ $v_x = a_x t$

$$\text{speed} = \sqrt{v_x^2 + v_y^2} =$$

$$= \sqrt{(-gt)^2 + (a_x t)^2}$$

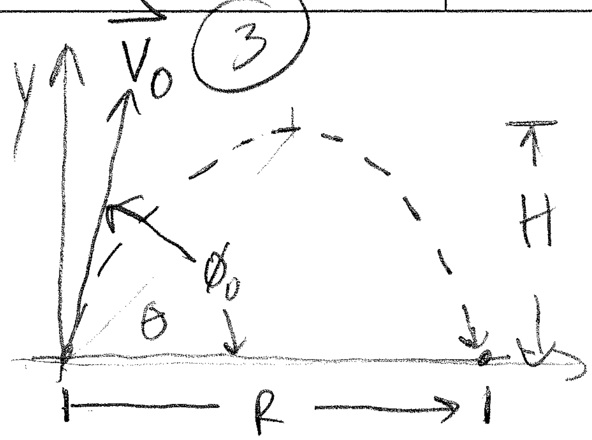
$$|\vec{v}| = \sqrt{g^2 + a_x^2}\ t \qquad \text{at} \quad t = \sqrt{\frac{2h}{g}}$$

$$|\vec{v}|_{\text{hit ground}} = \sqrt{g^2 + a_x^2} \cdot \sqrt{\frac{2h}{g}}$$

$$|\vec{v}|_{\text{hit ground}} = \sqrt{\left(g + \frac{a_x}{g} a_x\right) 2h}$$

$$|\vec{v}|_{\text{hit ground}} = \sqrt{\left(9{,}8 + \frac{1{,}2}{9{,}8} \cdot 1{,}2\right) \cdot 2 \cdot 39 \cdot \frac{m}{s^2} \cdot m}$$

$$= 27{,}9\ m/s$$



(a)

$$x = v_0 \cos\phi_0 \, t$$

$$y = v_0 \sin\phi_0 \, t - \frac{1}{2} g t^2$$

$$= \frac{x}{v_0 \cos\phi_0}\left(v_0 \sin\phi_0 - \frac{1}{2} g \frac{x}{v_0 \cos\phi_0}\right)$$

$y = 0$: $x = 0$, or, $v_0 \sin\phi_0 - \frac{1}{2} g \frac{R}{v_0 \cos\phi_0} = 0$

$$R = \frac{2 v_0^2}{g} \sin\phi_0 \cos\phi_0$$

$$R = \frac{v_0^2}{g} \sin(2\phi_0)$$

the time t_R at which the projectile reaches R is:

$$v_0 \cos\phi_0 \, t_R = R = \frac{v_0^2}{g} \sin(2\phi_0)$$

$$t_R = \frac{v_0^2}{g} \frac{2 \sin\phi_0 \cancel{\cos\phi_0}}{v_0 \cancel{\cos\phi_0}}$$

$$= 2 \frac{v_0}{g} \sin\phi_0$$

So the time where the trajectory achieves the maximum height is $\frac{1}{2} t_R = \frac{v_0}{g} \sin\phi_0$

so $$H = v_0 \sin\phi_0 \cdot \frac{v_0}{g} \sin\phi_0 - \frac{1}{2} g \frac{v_0^2}{g^2} \sin^2\phi_0$$

$$= \frac{1}{2} \frac{v_0^2}{g} \sin^2\phi_0$$

so, $$\frac{H}{R} = \frac{\frac{1}{2}\frac{\cancel{v_0^2}}{\cancel{g}}\sin^2\phi_0}{\frac{\cancel{v_0^2}}{\cancel{g}}\sin(2\phi_0)}$$

$$\boxed{\frac{H}{R} = \frac{1}{2}\frac{\sin^2\phi_0}{2\sin\phi_0\cos\phi_0} = \frac{1}{4}\frac{\sin\phi_0}{\cos\phi_0} = \frac{1}{4}\tan\phi_0}$$

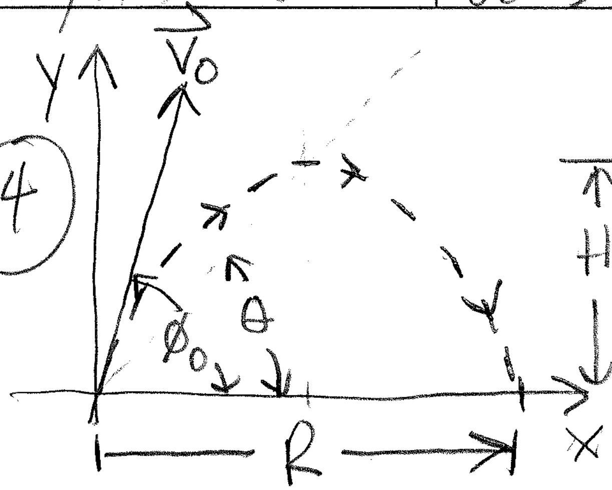
(b) $$\frac{H}{R} = \frac{1}{4}\tan\phi_0 = 1$$

$$\tan\phi_0 = 4$$

$$\boxed{\begin{aligned}\phi_0 &= \tan^{-1}(4) = 1.33 \text{ radians} \\ &= 76.0^\circ\end{aligned}}$$

TOPS

4



(a)

$$x = v_0 \cos\phi_0 \, t$$

$$y = v_0 \sin\phi_0 \, t - \frac{1}{2} g t^2$$

$$= \frac{x}{v_0 \cos\phi_0}\left(v_0 \sin\phi_0 - \frac{1}{2} g \frac{x}{v_0 \cos\phi_0}\right)$$

highest point is when $\frac{dy}{dx} = 0$, and $x = x_m$

$$\frac{dy}{dx} = \frac{\sin\phi_0}{\cos\phi_0} - g\frac{x_m}{v_0^2 \cos^2\phi_0} = 0$$

$$x_m = \frac{R}{2} = \frac{v_0^2}{g}\frac{\sin\phi_0}{\cancel{\cos\phi_0}}\cos^{\cancel{3}}\phi_0 \; \cos\phi_0$$

$$= \frac{v_0^2}{g}\sin\phi_0 \cos\phi_0$$

$$H = \frac{x_m}{v_0 \cos\phi_0}\left(v_0 \sin\phi_0 - \frac{1}{2} g \frac{x_m}{v_0 \cos\phi_0}\right)$$

$$= \frac{\cancel{v_0^2}\, v_0 \sin\phi_0 \cos\phi_0}{g \cancel{v_0} \cos\phi_0}\left(v_0 \sin\phi_0 - \frac{1}{2} g \frac{\cancel{v_0^2}\, v_0 \sin\phi_0 \cancel{\cos\phi_0}}{v_0 g \cancel{\cos\phi_0}}\right)$$

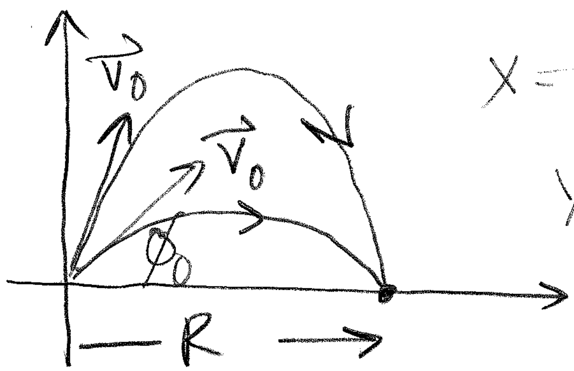
$$H = \frac{v_0^2}{2g}\sin^2\phi_0$$

and so, $\tan\theta = \frac{H}{x_m} = \frac{H}{(R/2)} = \dfrac{\frac{v_0^2}{2g}\sin^2\phi_0}{\frac{v_0^2}{g}\sin\phi_0\cos\phi_0}$

$$\boxed{\tan\theta = \frac{1}{2}\frac{\sin\phi_0}{\cos\phi_0} = \frac{1}{2}\tan\phi_0}$$

(b) $\tan\theta = \frac{1}{2}\tan(45^\circ) = \frac{1}{2}\tan\left(\frac{\pi}{4}\right) = \frac{1}{2}$, $\theta = 0.464$ radians $= 26.6^\circ$

(5).



Keep $|\vec{v}_0|$ same,

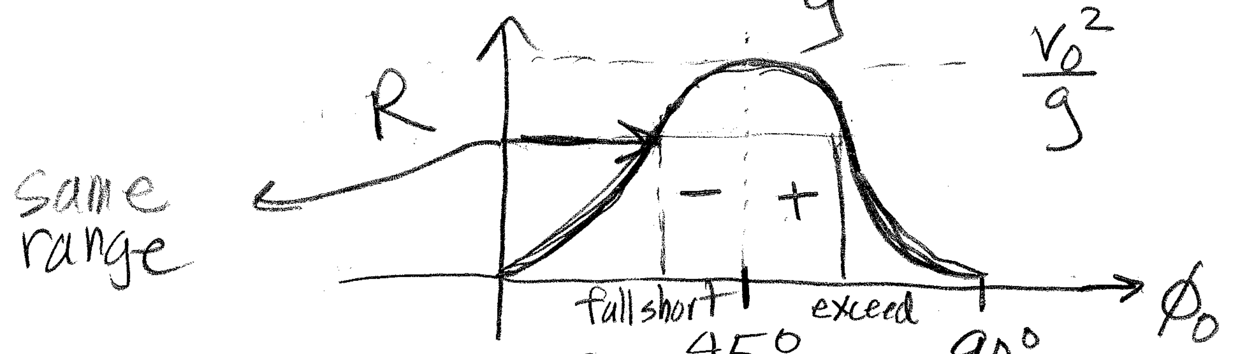
$$x = v_{0x} t = v_0 \cos\phi_0 \, t$$

$$y = v_0 \sin\phi_0 \, t - \frac{1}{2} g t^2$$

$$= \underbrace{\left(\frac{x}{v_0 \cos\phi_0}\right)}_{\substack{0 \\ \text{take off}}} \underbrace{\left(v_0 \sin\phi_0 - \frac{1}{2} g \left(\frac{x}{v_0 \cos\phi_0}\right)\right)}_{\text{landing}}$$

$$R = x = \frac{2v_0^2}{g} \sin\phi_0 \cos\phi_0$$

$$R = \frac{v_0^2}{g} \sin(2\phi_0)$$



$= \pi/4$ radians $= \pi/2$ radians

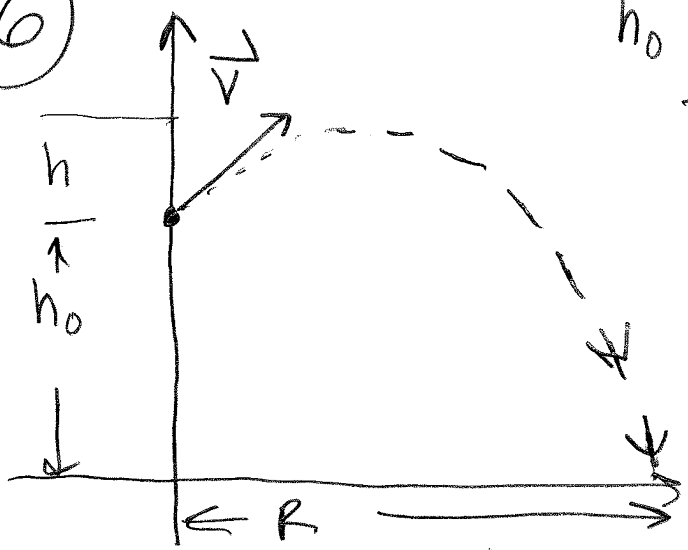
maximum

$\sin(2\phi_0)$ is symmetric about 45°, falling short, exceeding by same amount gets same range

(b), $\phi_0 = \frac{1}{2} \sin^{-1}\left(\frac{gR}{v_0^2}\right) = \frac{1}{2} \sin^{-1}\left(\frac{9.8 \cdot 20}{30^2}\right) = \frac{1}{2}\begin{pmatrix} 0.220, \\ 2.922 \end{pmatrix}$

$= 0.110, 1.461 = 6.29^\circ, 83.71^\circ$

(6)



$h_0 = 9.1$ m

$\vec{V} = V_x \hat{i} + V_y \hat{j}$

$V_x = 7.6$ m/s

$V_y = 6.1$ m/s

(a) maximum height...

(i) find time of $V_y = 0$

$$V_y - gt = 0$$

$$t = \frac{V_y}{g}$$

$$\boxed{\begin{aligned} h_0 + h &= h_0 + V_y t - \tfrac{1}{2} g t^2 \\ &= h_0 + \frac{V_y^2}{g} - \frac{1}{2} g \frac{V_y^2}{g^2} = h_0 + \frac{1}{2}\frac{V_y^2}{g} \\ &= 9.1\text{m} + \frac{1}{2}\frac{6.1^2 \text{ m}^2/\text{s}^2}{9.8 \text{ m/s}} = 11.0 \text{ m} \end{aligned}}$$

(ii) $\cancel{0}\ \cancel{V^2} - V_y^2 = -2gh$

$$h = \frac{V_y^2}{2g} = \frac{1}{2}\frac{6.1^2 \text{ m}^2/\text{s}^2}{9.8} = 1.90 \text{ m}$$

$$\boxed{h_0 + h = h_0 + \frac{1}{2}\frac{V_y^2}{g} = 9.1 + 1.9 \text{ m} = 11.0 \text{ m}}$$

(b) find total time... time to top is $t_1 = V_y/g$. Let t_2 = time to fall...

$$\frac{1}{2} g t_2^2 = h_0 + h = h_0 + \frac{1}{2}\frac{V_y^2}{g}$$

$$t_2 = \left[\frac{2}{g}\left(h_0 + \frac{1}{2}\frac{v_y^2}{g}\right)\right]^{1/2} = \left[\frac{2h_0}{g} + \frac{v_y^2}{g^2}\right]^{1/2}$$

so,

$$t_{tot} = t_1 + t_2$$

$$= \frac{v_y}{g} + \sqrt{\left(\frac{v_y}{g}\right)^2 + \frac{2h_0}{g}}$$

$$\boxed{\begin{aligned} R &= v_x t_{tot} = v_x\left(\frac{v_y}{g} + \sqrt{\left(\frac{v_y}{g}\right)^2 + \frac{2h_0}{g}}\right) \\ &= 7.6\,\text{m/s}\left[\frac{6.1\,\text{m/s}}{9.8\,\text{m/s}^2} + \sqrt{\left(\frac{6.1}{9.8}\right)^2 + \frac{2\cdot 9.1\,\text{m}}{9.8\,\text{m/s}^2}}\right] \\ R &= 7.6\,\text{m/s} \times (0.62\,\text{s} + 1.96\,\text{s}) = 19.6\,\text{m} \end{aligned}}$$

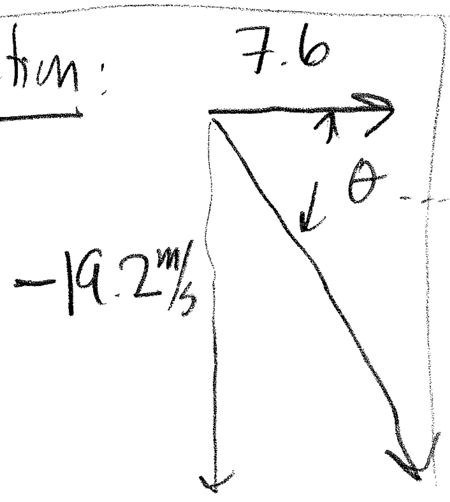
(c) v_x unchanged $= 7.6$ m/s

$$v_y = -g t_2 = -9.8\,\frac{\text{m}}{\text{s}^2} \cdot 1.96\,\text{s}$$

$$= -19.2\,\text{m/s}$$

$\boxed{\underline{\text{magnitude}} = \sqrt{v_x^2 + v_y^2} = 20.6\ \text{m/s}}$

<u>direction:</u>

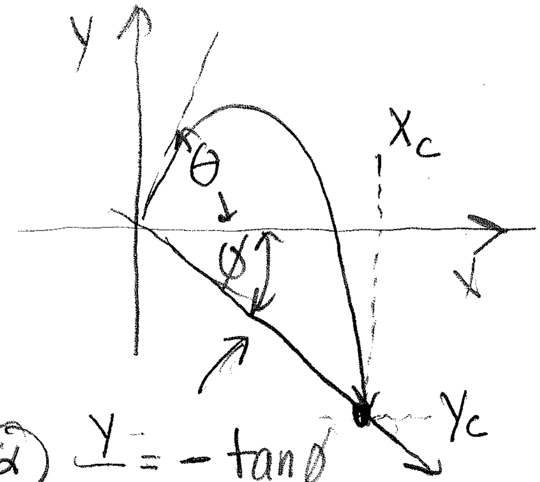


$$\theta = -\tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$= -\tan^{-1}\left(\frac{19.2\,\text{m/s}}{7.6\,\text{m/s}}\right)$$

$$= -1.194 \text{ radians}$$

$$= -68.4^\circ$$



trajectory:

$$x = v_{0x} t = v_0 \cos\theta \, t$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$= v_0 \sin\theta \, t - \frac{1}{2} g t^2$$

(α) $\frac{y}{x} = -\tan\phi$

convenient: $t = \frac{x}{v_0 \cos\theta}$

$$y = t\left(v_0 \sin\theta - \frac{1}{2} g t\right)$$

(β) $$y = \frac{x}{v_0 \cos\theta}\left(v_0 \sin\theta - \frac{1}{2}\frac{g x}{v_0 \cos\theta}\right)$$

(x_c, y_c) simultaneously solves both (α) and (β), so,

$$y_c = -\cancel{x_c} \tan\phi = \frac{\cancel{x_c}}{v_0 \cos\theta}\left(v_0 \sin\theta - \frac{1}{2} g \frac{x_c}{v_0 \cos^2\theta}\right)$$

$$-\tan\phi = \tan\theta - \frac{1}{2}\frac{g x_c}{v_0^2 \cos^2\theta}$$

$$\frac{1}{2}\frac{g x_c}{v_0^2 \cos^2\theta} = \tan\theta + \tan\phi$$

$$x_c = \frac{2 v_0^2 (\tan\theta + \tan\phi)\cos^2\theta}{g}$$

$$= \frac{2 v_0^2}{g}\left(\frac{\sin\theta}{\cos\theta}\cos^2\theta + \tan\phi \cos^2\theta\right)$$

$$x_c = \frac{2 v_0^2}{g}\left(\underbrace{\sin\theta \cos\theta}_{\frac{1}{2}\sin 2\theta} + \tan\phi \cos^2\theta\right)$$

Maximize x_c w/r to θ

$$\frac{dx_c}{d\theta} = \frac{2 v_0^2}{g}\left(\cos 2\theta - 2 \tan\phi \cos\theta \sin\theta\right) = 0$$

$$\cos 2\theta - \tan\phi \cdot \sin 2\theta = 0$$

$$\sin 2\theta \tan\phi = \cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta = \frac{1}{\tan\phi}$$

$$\boxed{\begin{aligned} \theta &= \frac{1}{2}\left[\tan^{-1}\left(\frac{1}{\tan\phi}\right)\right] \\ \tan^{-1}\left(\frac{1}{\tan\phi}\right) &= \frac{\pi}{2} - \phi \\ \theta &= \frac{\pi}{4} - \frac{\phi}{2} = 45^\circ - \frac{1}{2}\phi \end{aligned}}$$

clue: $\phi = 60^\circ$, $\theta = 45^\circ - 30^\circ = 15^\circ$

Note, works for $-90^\circ < \phi < 90^\circ$