Physics 125 Problem Set 6

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due Friday, May 15 in class

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1. One consequence of our work in class on spin quantized along an arbitrary axis is that you can easily now get the eigenvalues and eigenvectors of an arbitrary Hermitian 2 by 2 Hamiltonian. All you have to do is re-write the general Hamiltonian in the following way:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{12}^* & H_{22} \end{bmatrix} = \eta_0 \mathbf{I} + \eta_1 \left(\sin\theta_H \cos\phi_H \boldsymbol{\sigma}_x + \sin\theta_H \sin\phi_H \boldsymbol{\sigma}_y + \cos\theta_H \boldsymbol{\sigma}_z \right).$$

   Note that the 4 real parameters H_{11}, H_{22}, $\mathrm{Re}(H_{12})$, and $\mathrm{Im}(H_{12})$ are replaced, on the right hand side of the equation, by new parameters η_0, η_1, θ_H, and ϕ_H, and bold-faced symbols represent the identity and the 3 Pauli matrices.

   (a) Solve for the four new parameters in terms of the old parameters.

   (b) What are the eigenvalues in terms of η_0 and η_1, and hence in terms of the H's? Don't solve the characteristic equation (unless you want to check) just write down the eigenvalues.

   (c) Repeat for the eigenvectors, adding in the dependence on θ_H and ϕ_H.

2. Griffiths 4.19

3. Griffiths 4.20

4. Griffiths 4.21

5. Griffiths 4.38

6. Griffiths 5.1

7. Griffiths 5.6
