Physics 125 Problem Set 2

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due Friday, April 17 in class

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1. In this problem, we apply your knowledge of spin-1/2 to probe the nature of the very lightest mesons: those formed by $u\overline{u}$ and $d\overline{d}$. The connection with spin-1/2 is simply: both are 2-state quantum systems. What is unusual and important in the $u\overline{u}$-$d\overline{d}$ system is the influence of annihilating transitions, for example, from $u\overline{u}$ to $d\overline{d}$, on the formation of the eigenstates. In this problem, we'll neglect everything but the strong interaction, and we'll assume that the strong interaction treats u and d quarks identically... this is known as 'isospin' invariance.

   (a) The 'normal' kind of energy you are used to, for example, from the hydrogen atom, is caused by processes that don't destroy the quarks. These are described by Feynman diagrams where the fermion lines continue without breaks from the beginning to the end of the process. The types of contributions to these diagrams include all the 'condensed enery' of the gluon field. Let's call the portions of the strong Hamiltonian that contribute to this energy H_1. The key point is (letting A be a positive real number):

   $$\begin{aligned} A &= \langle u\overline{u}|H_1|u\overline{u}\rangle = \langle d\overline{d}|H_1|d\overline{d}\rangle \\ 0 &= \langle u\overline{u}|H_1|d\overline{d}\rangle = \langle d\overline{d}|H_1|u\overline{u}\rangle \end{aligned}$$

   (b) The 'annihilation' contribution, caused by matter-antimatter annihilation into 'the glue'. That is, $u\overline{u} \to$ glue, but, then, what does the glue do? The glue itself only has 2 options: it comes back as $u\overline{u}$ or $d\overline{d}$, and assuming isospin invariance, the quantum mechanical amplitudes must be equal. Let's call the portions of the strong Hamiltonian that contribute to this energy H_2. Then, letting B be a positive real number:

   $$\begin{aligned} B &= \langle u\overline{u}|H_2|u\overline{u}\rangle = \langle d\overline{d}|H_2|d\overline{d}\rangle \\ B &= \langle u\overline{u}|H_2|d\overline{d}\rangle = \langle d\overline{d}|H_2|u\overline{u}\rangle \end{aligned}$$

   (c) Write down the 2 by 2 matrix that represents the total strong Hamiltonian $H_1 + H_2$, and find its eigenvalues and eigenvectors.

   (d) One of the eigenstates is the π^0, with a rest energy of 135 MeV, and the other is the η, with a rest energy of 548 MeV. Write down relation between the $|\pi^0\rangle$ and $|\eta\rangle$ states in terms of the $|u\overline{u}\rangle$ and $|d\overline{d}\rangle$ states. Determine the numerical value of the constants A and B.

   (e) Why is is the mass of the *charged* pion, 140 MeV, approximately equal to that of the π^0? (think about presence/absence of annihilation energy contributions).

   (f) Suppose, somehow, you produce at $t = 0$, a $u\overline{u}$ state.

      i. If you now measure its rest energy, what values do you get, with what probabilities?

      ii. Find the expression describing the probability of finding the state to be $u\overline{u}$ at all times after $t = 0$, and make a plot of that probability as a function of time (in suitable numerical units of time).
