Now $\mathcal{M}$ might depend on direction of $\vec{p}_3 \ldots$ $\vec{p}_1 \cdot \vec{p}_3$ around

$$d^3|\vec{p}_3| = |\vec{p}_3|^2 \, d|\vec{p}_3| \, d\Omega$$

$\Rightarrow$ give up full $\sigma$, go for

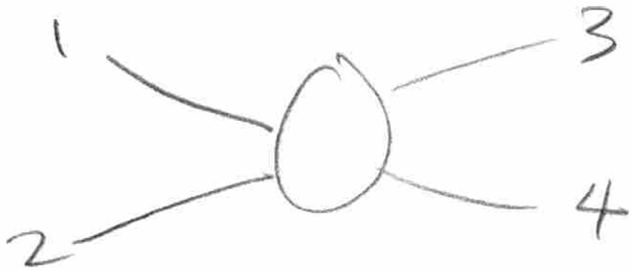
$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \left[\frac{|\vec{p}_3| = |\vec{p}_f|}{|\vec{p}_1| = |\vec{p}_i|}\right]$$

dimensions...

$$\ell^2 = (\hbar c)^2 \to (\cancel{E} \cdot \ell)^2 \frac{1}{\cancel{E}^2} \cdot |\mathcal{M}|^2$$

$|\mathcal{M}|^2$ dimensionless

4 lines




$$\dim \mathcal{M} \to p^{4-n}$$

$\uparrow$ # lines

$$\frac{d\sigma}{d\Omega} = \left|\frac{b}{\sin\theta}\frac{db}{d\theta}\right| = \frac{R\cos\frac{\theta}{2}\,\frac{R}{2}\sin\frac{\theta}{2}}{\sin\theta}$$

$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4} \qquad \underline{\underline{\text{CONSTANT}}}$$

$$\int d\Omega\left(\frac{d\sigma}{d\Omega}\right) = \frac{R^2}{4}\cdot(4\pi\ sr)$$

$$= \pi R^2$$

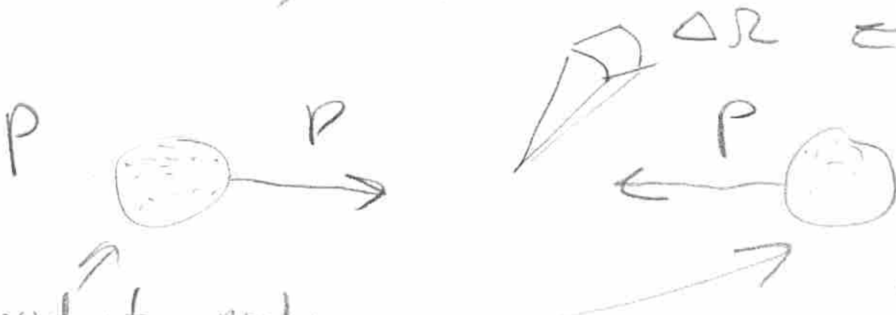
"Luminosity" $\underset{\frac{\text{part}}{cm^2\ s}}{\phi} \longrightarrow$

$A \quad \rightarrow \leftarrow z$

$$\text{Rate} = \underset{cm^2}{A}\ \underset{\frac{\text{particles}}{cm^2\ s}}{\phi}\ \underset{cm^2}{d\sigma} \cdot \underset{\frac{1}{cm^3}}{\rho_N} \cdot \underset{cm}{z}$$

$$= \underbrace{(A\phi\rho_N z)}_{\text{"Luminosity" } \mathcal{L}}\ d\sigma$$

$1/cm^2\,s$ like flux, not flux




$\leftarrow$ Rate $\simeq \mathcal{L}\frac{d\sigma}{d\Omega}\Delta\Omega$

cloud of protons

LHC: 16 μm, 7.5 cm

$10^{34}\ cm^2/s$, 25 ns

$\Rightarrow$ do all the $p_j^0$ integrals, use the $\delta$-functions that enforce masses. Only particles in final state in integral

$$\sigma = \frac{S\hbar^2 c}{64\pi^2(E_1+E_2)|\vec{p}_1|}$$

$$\int |\mathcal{M}|^2 \frac{\delta^4(p_1+p_2-p_3-p_4)}{\sqrt{|\vec{p}_3|^2+(m_3c)^2}\sqrt{|\vec{p}_4|^2+(m_4c)^2}} d^3p_3\, d^3p_4$$

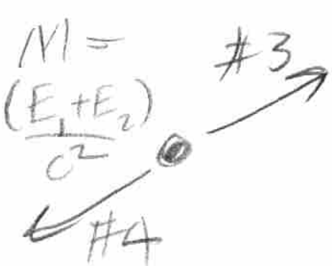
$$\delta^4(p_1+p_2-p_3-p_4)$$

$$= \underbrace{\delta\left(\frac{E_1+E_2}{c}-p_3^0-p_4^0\right)}_{\text{you know}} \underbrace{\delta^3(\vec{p}_3+\vec{p}_4)}_{\text{enables } d^3p_4 \text{ integral to be done!}}$$

$$p_3^0 = \frac{E_3}{c} = \sqrt{m_3^2c^2+|\vec{p}_3|^2}$$

$$p_4^0 = \frac{E_4}{c} = \sqrt{m_4^2c^2+|\vec{p}_4|^2}$$

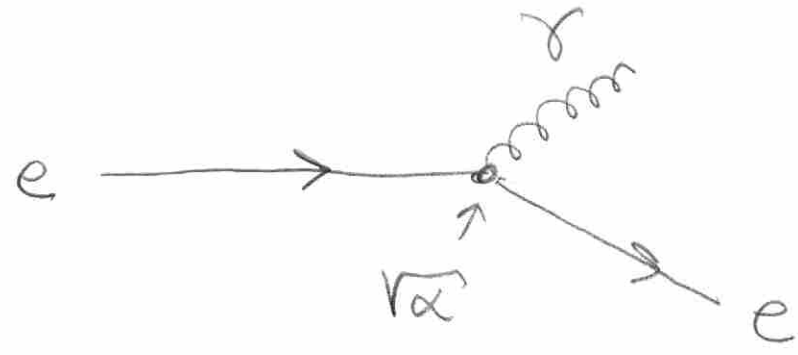
$$\vec{p}_4 = -\vec{p}_3$$



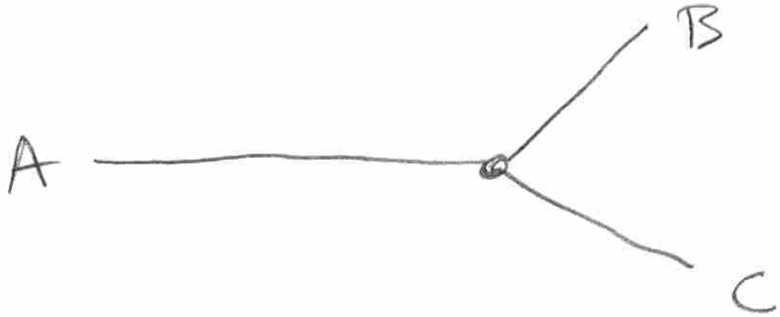

$$|\vec{p}_3|^2 = |\vec{p}_4|^2 = \frac{\lambda(M^2, m_3^2, m_4^2)\,c^2}{M^2}$$

Same as before.. (not surprising).

# ABC Theory




$e \to e\gamma$
energy
momentum




$A = \bar{A}$ $\quad$ $B = \bar{B}$ $\quad$ $C = \bar{C}$

$m_A$ $\quad$ $m_B$ $\quad$ $m_C$

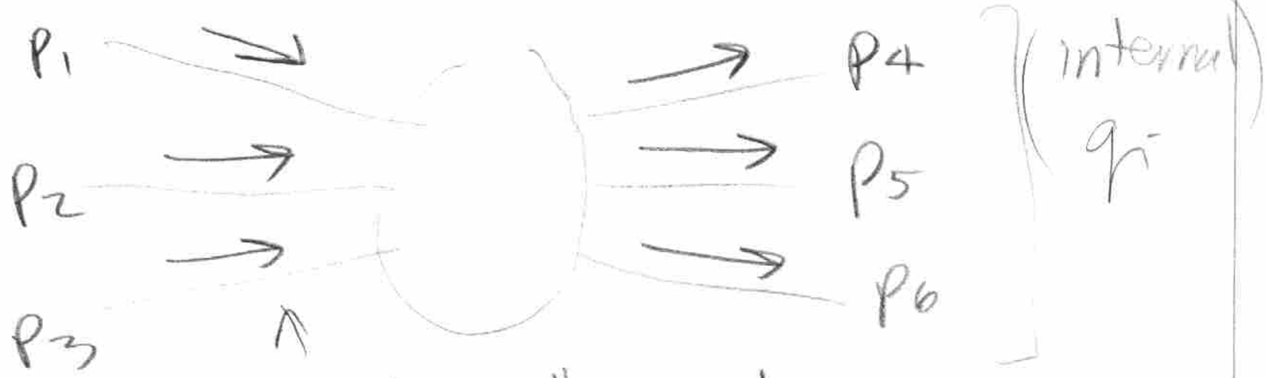
$m_A > m_B + m_C$ decay of the A possible

(no arrows)

## To Compute $\mathcal{M}$

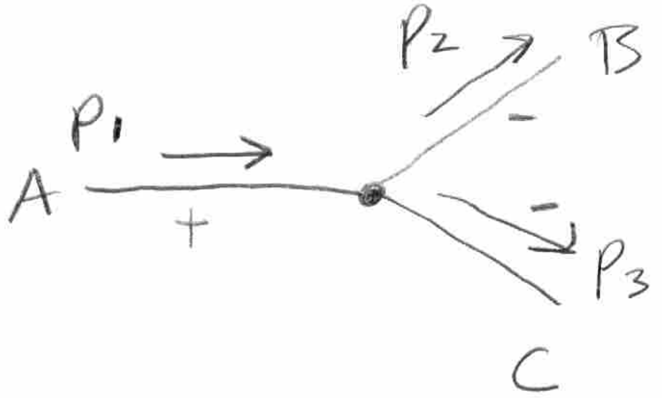
1. Label

4-momentum




incoming $\quad$ $\wedge$ means "positive" $\quad$ out going

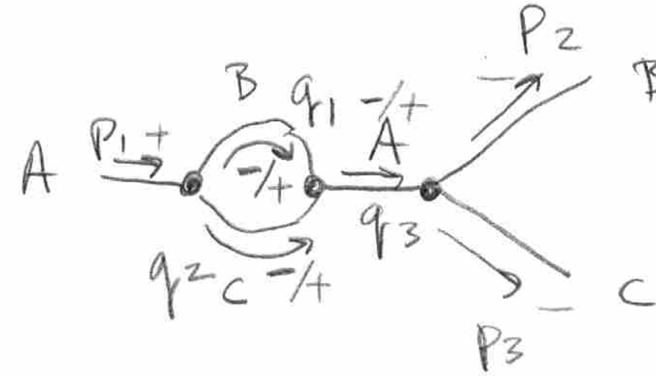
(α)




$-ig$

no internal line!

(β)




$(-ig)^3$ ] more neglected

2. Vertex Factor $-ig$

3. Propagators

"Internal Line" $\dfrac{i}{q_i^2 - m_j^2c^2} \neq \dfrac{i}{0}$

virtual

(β): $\left(\dfrac{i}{q_1^2 - m_B^2c^2}\right)\cdot\left(\dfrac{i}{q_2^2 - m_C^2c^2}\right)\left(\dfrac{i}{q_3^2 - m_A^2c^2}\right)$

4. Vertex Momentum Conservation

$(2\pi)^4\delta(p_1 - p_2 - p_3)$ (α)

$$\left.\begin{array}{c}(2\pi)^4\delta^4(p_1-q_1-q_2)(2\pi)^4\delta^4(q_1+q_2-q_3)\\ (2\pi)^4\delta^4(q_3-p_2-p_3)\end{array}\right\}\ \beta$$

5 Integrate over the $q$'s (internal)

$\frac{1}{(2\pi)^4}d^4q_j$     0 integrals $\alpha$

3 integrals

6 After integrals, there is always a $\delta^4(p_1+p_2+\dots-p_n)$

<u>Cancel</u> <u>one</u> $(2\pi)^4\delta^4(\quad)$ in $\mathcal{M}$. Multply by $i$. That is $\mathcal{M}$

$\alpha$    $-ig\times\cancel{(2\pi)^4\delta^4(p_1-p_2-p_3)}\times i$

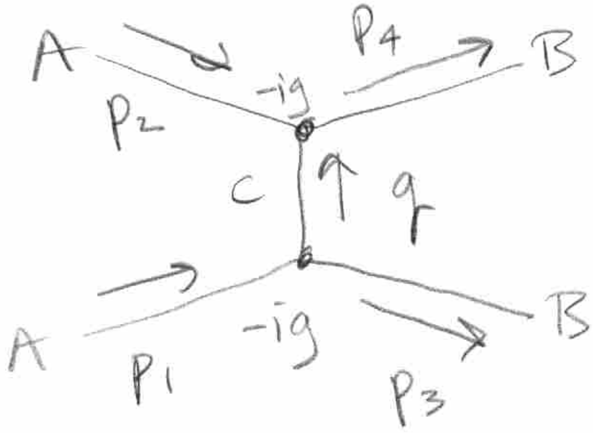
cancel

$= g$ !

$$\Gamma = \frac{g^2|\vec{p}|}{8\pi\hbar m_A^2 c}, \quad \tau = \frac{8\pi\hbar m_A^2 c}{g^2|\vec{p}|}$$

$$|\vec{p}|^2 = \frac{\lambda(m_A^2, m_B^2, m_C^2)}{4m_A^2}c^2$$

## $A + A \to B + B$




vertex factor

$$(-ig)^2 = -g^2$$

propagator

$$\frac{i}{q^2 - m_c^2 c^2}$$

δ - functions

$$(2\pi)^4 \delta^4(p_1 - q - p_3) \quad (2\pi)^4 \delta^4(p_2 + q - p_4)$$

$$-ig^2 (2\pi)^4 \int \frac{1}{q^2 - m_c^2 c^2} \delta^4(p_1 - q - p_3) \delta^4 \underbrace{(p_2 + q - p_4)}$$

$$p_2 + q - p_4 = 0$$

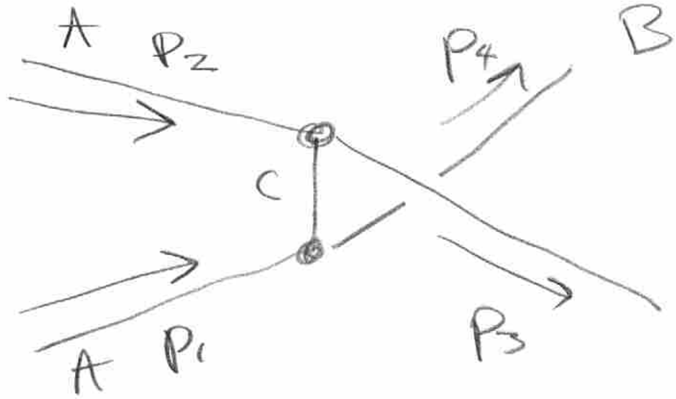
$$q = p_4 - p_2$$

$$= \frac{-ig^2 (2\pi)^4 \ \delta^4(p_1 + p_2 - p_3 - p_4)}{(p_4 - p_2)^2 - m_c^2 c^2}$$

cancel $\delta^4$, multiply by $i$,

$$\mathcal{M}_1 = \frac{g^2}{(p_4 - p_2)^2 - m_c^2 c^2}$$

## Indistinguishable Amplitude



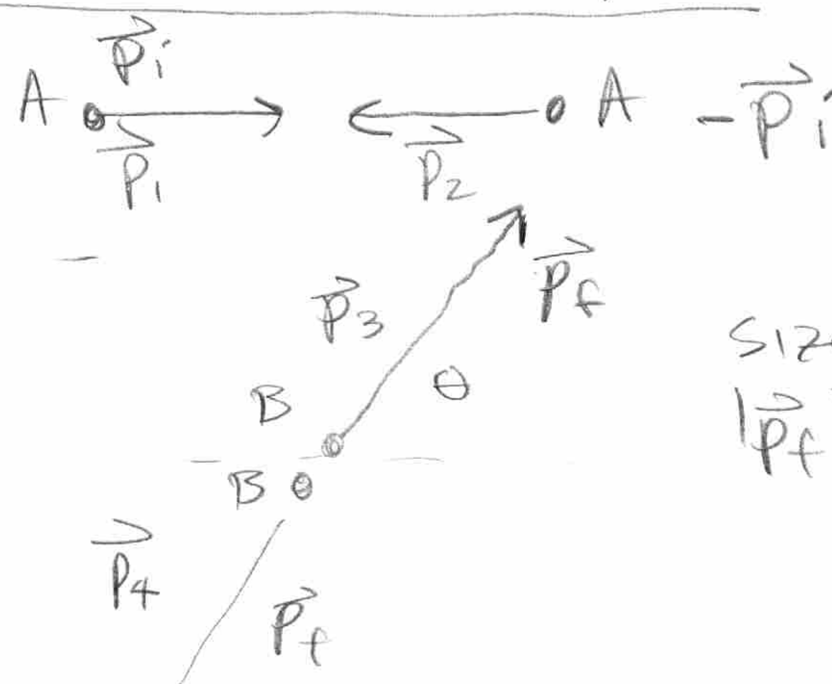

$\mathcal{M}_2$

Swap $p_3$ + $p_4$ in last amplitude

$$\mathcal{M}_1 + \mathcal{M}_2 = \underbrace{\frac{g^2}{(p_4 - p_2)^2 - m_c^2 c^2} + \frac{g^2}{(p_3 - p_2)^2 - m_c^2 c^2}}_{}$$

add <u>amplitudes</u>
indistinguishable

## Center of Mass system




size
$|\vec{p}_f| \lesseqgtr |\vec{p}_i|$?

$\vec{p}_i$ ?  $|\vec{p}_i|^2 = \dfrac{\lambda((2E_A/c)^2, m_A^2, m_A^2)}{4(2E_A/c)^2} c^2$

$|\vec{p}_f| = \dfrac{\lambda((2E_A/c)^2, m_B^2, m_B^2)}{4(2E_A/c)^2} c^2$

$(p_4 - p_2)^2 - \underbrace{m_c^2 c^2}_{0} = ?$  Take $m_c = 0$

$m_A = m_B = m$

$|\vec{p}_i| = |\vec{p}_f|$

$= p_4^2 + p_2^2 - 2 p_2 \cdot p_4$    $E_1 = E_2 = E_3 = E_4 = E$

$= m_B^2 c^2 + m_A^2 c^2 - 2(\underbrace{E_2 E_4}_{E^2} - \underbrace{\vec{p}_2 \cdot \vec{p}_4}_{|\vec{p}|^2 \cos\theta})$

$\underbrace{2(E^2 - m^2 c^2)}_{2|\vec{p}|^2} + 2|\vec{p}|^2 \cos\theta$

$(p_4 - p_2)^2 - m_c^2 c^2 = -2|\vec{p}|^2 (1 - \cos\theta)$

$(p_3 - p_2)^2 - m_c^2 c^2 = -2|\vec{p}|^2 (1 + \cos\theta)$

$\mathcal{M}_1 + \mathcal{M}_2 = \dfrac{g^2}{-|\vec{p}|^2 2} \left( \dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta} \right)$

$\left( \dfrac{1+\cos\theta + 1 - \cos\theta}{1 - \cos^2\theta} \right) = \dfrac{2}{\sin^2\theta}$

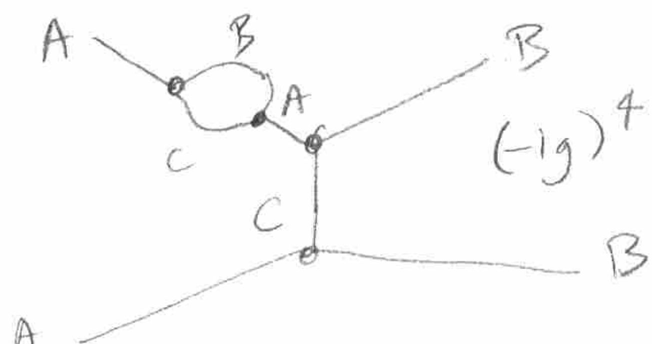
TOPS
95590

$$\mathcal{M}_1 + \mathcal{M}_2 = -\frac{g^2}{|\vec{p}|^2 \sin^2\theta}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{(S=\frac{1}{2})}{4E^2} \frac{g^4}{|\vec{p}|^4 \cdot \sin^4\theta}$$

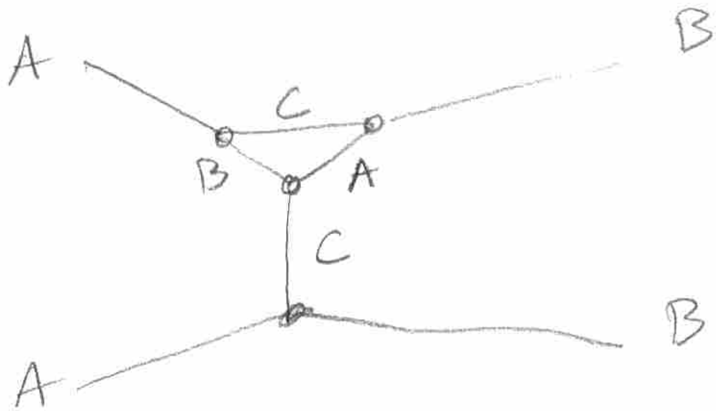
$$= \frac{1}{2}\left(\frac{\hbar c g^2}{16\pi E |\vec{p}|^2 \sin^2\theta}\right)$$

"Higher Order" Diagrams

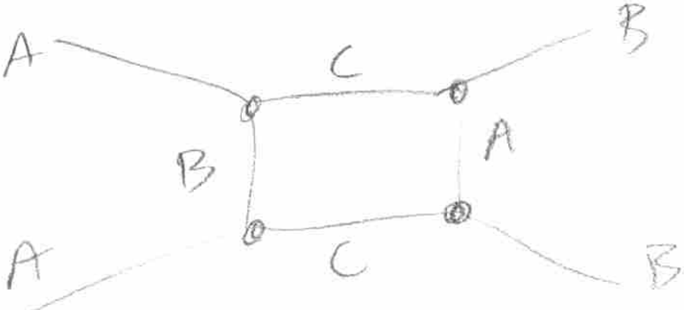



$(-ig)^4$

○ could go in 5 spots




2 of these




1 of these

Problem: contributions $= \infty$!

Luckily $\underline{\ln \infty}$

(an integral like $\frac{1}{q^4} q^3 dq$).

"Regularization"

$$\frac{1}{(q^2)^2} \rightarrow \frac{1}{q^2 - M^2c^2}$$

$$M \gg q.$$

"New Physics" ??

when all the integrals done...

$$m_{physical} = m_{bare} + \delta m$$

observed

if $\infty$

say that $m_{phys} - \infty$

$$g_{physical} = g_{bare} + \delta g$$

etc