Golden Rule

Rate, Cross Section

"Amplitude" $\Leftrightarrow$ "Matrix Element"

Dynamics (Feynman Diagrams)

"Phase Space" $\Leftrightarrow$ "Density of Final States"

Kinematics

Idea: [six dimensional config space

6-d volume $\rightarrow$ $\dfrac{\int (dx\,dy\,dz)(dp_x\,dp_y\,dp_z)}{(2\pi\hbar)^3}$

Planck!

Black body!

Time $\rightarrow$ special, controls

Rates, Energy Conservation

$$\overset{m_1}{1} \longrightarrow \overset{m_2}{2} + \overset{m_3}{3} + \overset{m_4}{4} + \ldots\ldots \overset{m_n}{n}$$

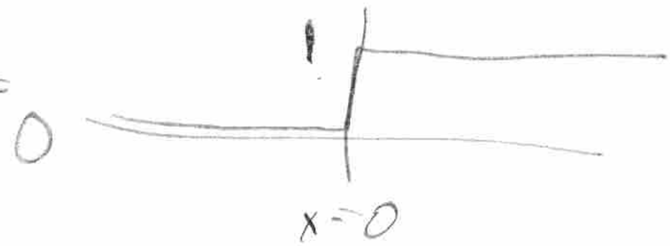
decay rate
in #1 rest frame.

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4 (\underbrace{p_1 - \sum p_i}_{\text{4 momenta!}})$$

$$\times \prod_{j=2}^{n} 2\pi\, \delta(p_j^2 - m_j c^2)\, \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

4(n − 1) integrals!

$\theta(x) =$



Idea: uncertainty principle allows "exploration" of "wave" kinematics. "trap" waves in δ-functions (limiting process)

All the integrals... just "counting" available states.

$$\delta(p^2 - m^2c^2) = \delta\left(\left(\frac{E}{c}\right)^2 - |\vec{p}|^2 - m^2c^2\right)$$

aka p^0

$$\underbrace{\delta(x^2 - a^2)} = \frac{1}{2a}\left[\delta(x-a) + \delta(x+a)\right]$$

(A.9 – A.13)

- $x^2 - a^2 = 0$
  $x = \pm a$
- $1/\text{length}^2$ area = 1

$$a = |\vec{p}|^2 + m^2c^2$$

$$\Gamma = \frac{S}{2\hbar m_1}\int |\mathcal{M}|^2 (2\pi)^4 \delta^4\left(p_1 - \sum_i p_i\right)$$
$$\times \prod_{j=2}^{n} \frac{1}{2\sqrt{|\vec{p}_j|^2 + m_j^2c^2}} \frac{d^3p_j}{(2\pi)^3}$$

(S: identical particle factor
$a \to b + b + c + c + c$ $\quad S = \frac{1}{2!}, \frac{1}{3!}$)

$\left((2\pi\delta), \left(\frac{dp}{2\pi}\right)\right)$

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2 particle decay $\qquad 1 \to 2 + 3$

$$\Gamma = \frac{S}{2\cdot 2^2 \hbar m_1} \frac{(2\pi)^4}{(2\pi)^6}$$
$$\int |\mathcal{M}|^2 \frac{\delta^4(p_1 - p_2 - p_3)}{\sqrt{|\vec{p}_2|^2 + m_2^2c^2}\sqrt{|\vec{p}_3|^2 + m_3^2c^2}}\, d^3p_2\, d^3p_3$$

$$\delta^4(p_1 - p_2 - p_3)$$
$$= \delta\left(m_1c - \sqrt{|\vec{p}_2|^2 + m_2^2c^2} - \sqrt{|\vec{p}_3|^2 + m_3^2c^2}\right)$$
$$\cdot\, \delta^3(\vec{p}_2 + \vec{p}_3) \quad \to \text{do } p_3 \text{ integral!}$$

$\vec{p}_2 = -\vec{p}_3$

$|\vec{p}_2|$ <u>you know</u>!

$$c^2|\vec{p}_2|^2 = \frac{m_1^4+m_2^4+m_3^4-2m_1^2m_2^2-2m_2^2m_3^2-m_1^2m_3^2}{4m_1^2}c^4$$

$$d^3p_2 = |\vec{p}_2|^2 \sin\theta \, d\theta \, d\phi \; d|\vec{p}_2|$$

<u>ASSUME</u>: $\mathcal{M}$ independent of angle

<u>must be true</u>

$$\int \sin\theta \, d\theta \, d\phi = 4\pi$$

$$\Gamma = \frac{S}{8\pi\hbar m_1}\int_0^\infty |\mathcal{M}(|\vec{p}_2|)|^2 \underbrace{\frac{\delta(m_1-\sqrt{\quad}-\sqrt{\quad})}{\sqrt{\quad}\cdot\sqrt{\quad}}}_{\text{just a } \delta\text{- function!}} |\vec{p}_2|^2 \, d|\vec{p}_2|$$

$$u(|\vec{p}_2|) = \sqrt{|\vec{p}_2|^2+m_2^2c^2}+\sqrt{|\vec{p}_2|^2+m_3^2c^2}$$

deal with $\delta(m_1 - u(|\vec{p}_2|))$

→ know the $|\vec{p}_2|$ value!

→ all the art in the functional form!

so

$$\Gamma = \frac{S}{8\pi\hbar m_1} \int_0^\infty |\mathcal{M}(|\vec{p}_2|)|^2 \frac{\delta(m_1 c - \sqrt{\quad} - \sqrt{\quad})}{\sqrt{\quad}\cdot\sqrt{\quad}} |\vec{p}_2|^2 d|\vec{p}_2|$$

$$= \frac{S}{8\pi\hbar m_1} \int_0^\infty |\mathcal{M}(|\vec{p}_2|)|^2 \underbrace{\delta(m_1 c - u)\frac{|\vec{p}_2|}{u}\, du}_{\text{now "changed" variables}}$$

$$u \to m_1 c$$

$$|\vec{p}_2| \to c^2 \frac{\lambda(m_1^2, m_2^2, m_3^2)}{4 m_1^2}$$

$$\boxed{\Gamma = \frac{S\, |\vec{p}_2|}{8\pi\hbar m_1^2 c} |\mathcal{M}|^2}$$

Quite remarkable ...

.. check dimensions ..

$$\frac{1}{\text{time}} = \frac{S}{8\pi} \frac{|\vec{p}_2|}{m_1 c} \left(\frac{1}{m_1 \hbar} \right) |\mathcal{M}|^2$$

$$\frac{E^2}{c^2} = (\text{momentum})^2$$

$$\frac{1}{\left(\frac{\text{Energy}}{c^2}\,\text{Energy}\right)\cdot\text{time}}$$

dim $\mathcal{M} \to$ momentum (2 particles in final state

add another particle

add: $\underbrace{\underbrace{\delta(p_j^2 - m_j c^2)\, d^4 p}_{\frac{1}{(\text{momentum})^2} \times (\text{momentum})^4}}_{(\text{momentum})^2}$

$\mathcal{M} \rightarrow$ down by one power

$\mathcal{M} \propto p^x \qquad x = a - m$

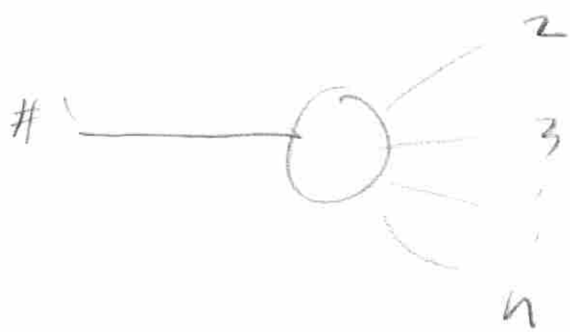
↑ # particles in final state.

when $m = 2,\ x = 1$

$1 = a - 2 \qquad a = 3$

$\mathcal{M} \propto p^{3-m}$ $\qquad m =$ # particles in final state.

or $\propto p^{4-n}$ $\qquad n =$ # particle lines.



① $\delta(f(x)) = 0$ when $f(x_i) = 0$

$\rightarrow |\vec{p}_2| =$ as above.

② $\int \delta(f(x))\, df = 1$

or $\underbrace{\int \delta(f(x)) \frac{df}{dx}}\, dx = 1$

$$\delta(x - x_i) = \delta(f(x)) \left.\frac{df}{dx}\right|_{x=x_i}$$

or $\delta(f(x)) = \dfrac{\delta(x - x_i)}{(df/dx)_{x=x_i}}$

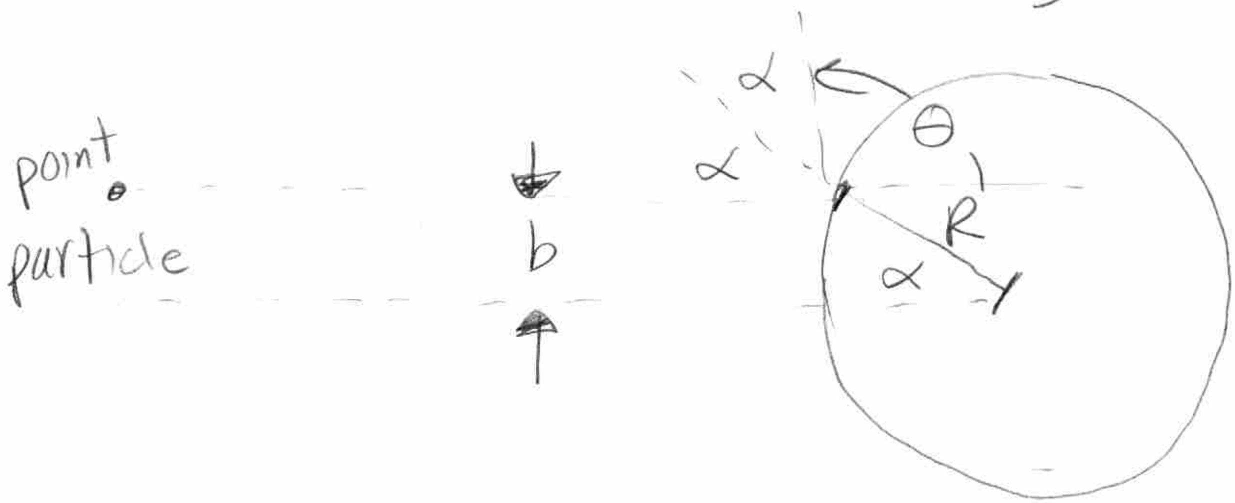
$$\frac{du}{d|\vec{p}_2|} = \frac{|\vec{p}_2|}{\sqrt{}} + \frac{|\vec{p}_2|}{\sqrt{}}$$

$$= \frac{|\vec{p}_2|\, u(|\vec{p}_2|)}{\sqrt{}\sqrt{}}$$

$$\frac{\delta(m_1 c - u)\, |\vec{p}_2|}{\sqrt{}\sqrt{}} = \delta(m_1 c - u) \frac{du}{d|\vec{p}_2|} \cdot \frac{1}{u}$$

Differential Cross Section

Hard sphere scattering (classical)

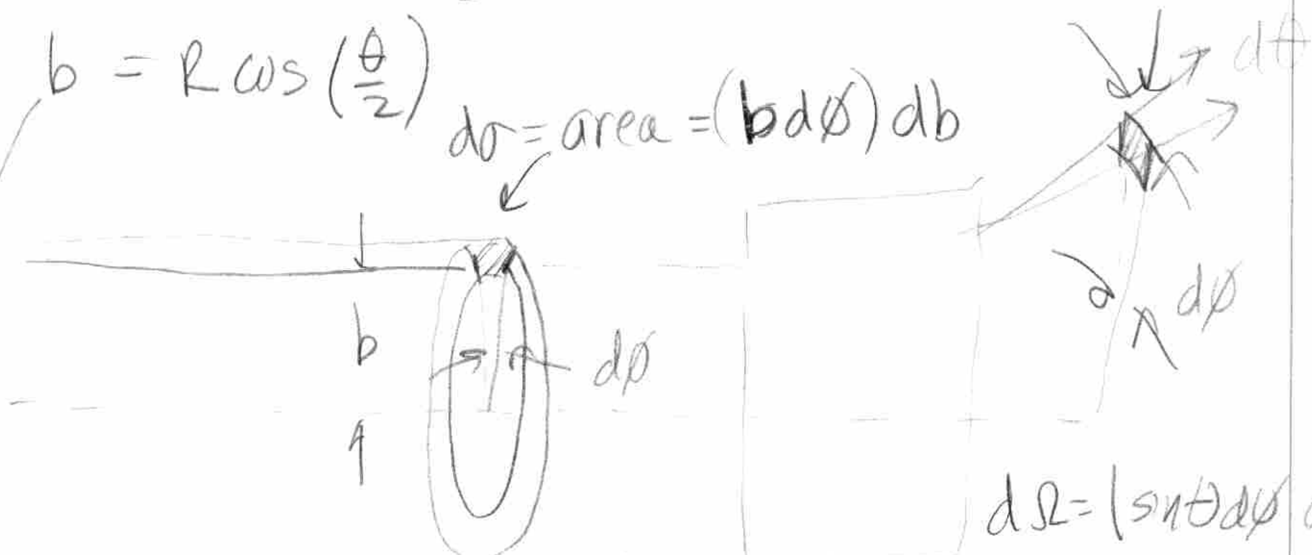


$$b = R\sin\alpha \qquad 2\alpha + \theta = \pi$$

$$= R\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) \qquad \alpha = \frac{\pi}{2} - \frac{\theta}{2}$$

$$b = R\cos\left(\frac{\theta}{2}\right)$$

$d\sigma$ = area = $(b\,d\phi)\,db$



$d\Omega = |\sin\theta\, d\phi\, d\theta|$

$$\frac{d\sigma}{d\Omega} = \left|\frac{b\,\cancel{d\phi}\,db}{\sin\theta\,\cancel{d\phi}\,d\theta}\right| = \left|\frac{b}{\sin\theta}\frac{db}{d\theta}\right|$$

$$db = -R\sin\left(\frac{\theta}{2}\right)\frac{d\theta}{2} \rightarrow \frac{db}{d\theta} = -\frac{R}{2}\sin\frac{\theta}{2}$$

Golden Rule for Scattering

$$1+2 \rightarrow 3+4+\ldots n$$

$$\sigma\left(\frac{1}{cm^2}\right) = \frac{S\hbar^2}{4\sqrt{(p_1\cdot p_2)^2-(m_1 m_2 c^2)^2}} \times$$

$$\int |\mathcal{M}|^2 (2\pi)^4 \overbrace{\delta^4(p_1+p_2-p_3+\ldots-p_n)}^{\text{conserve 4-mom}}$$
$$\prod_{j=3}^{n} (2\pi)\,\delta(p_j^2-m_j c^2)\,\Theta(p_j^0)\,\frac{d^4 p_j}{(2\pi)^4}$$

$$(p_1\cdot p_2)^2-(m_1 m_2 c^2)^2 = \text{"invariant"}$$

may want it in terms of quantities in a specific frame...

#1 CM frame

$$\vec{P}_1 \longrightarrow \bullet \longleftarrow \vec{P}_2 = -\vec{P}_1$$

$$(p_1+p_2)^2 = p^2 = M^2c^2 = \frac{(E_1+E_2)^2}{c^2}$$

$m_1^2 +$

$$Mc^2 = E_1+E_2$$

$$|\vec{P}_1|^2 = \frac{\lambda(M^2, m_1^2, m_2^2)}{4M^2}\, c^2 \quad \text{remember?}$$

$$(p_1 + p_2)^2 = P^2 = M^2 c^2$$

$$m_1^2 c^2 + m_2^2 c^2 + 2 p_1 \cdot p_2 = M^2 c^2$$

$$p_1 \cdot p_2 = \frac{1}{2}\left[M^2 - m_1^2 - m_2^2\right] c^2$$

$$(p_1 \cdot p_2)^2 - (m_1^2 m_2^2) c^4$$

$$= \frac{1}{4}\left[M^2 - m_1^2 - m_2^2\right]^2 c^4 - m_1^2 m_2^2 c^4$$

$$= \frac{1}{4}\left[M^4 + m_1^4 + m_2^4 - 2M^2 m_1^2 - 2M^2 m_2^2 + 2 m_1^2 m_2^2\right] c^4$$

$$- m_1^2 m_2^2 c^4$$

$$= \frac{1}{4} \lambda (M^2, m_1^2, m_2^2) c^4 = M^2 |\vec{p}_1|^2 c^2$$

$$\sqrt{(p_1 p_2)^2 - (m_1 m_2 c^2)^2} = M |\vec{p}_1| c$$

$$= \frac{(E_1 + E_2) |\vec{p}_1|}{c}$$

Different in frame where particle #2 is at rest..

$$\sqrt{(p_1 p_2)^2 - (m_1 m_2 c^2)^2} = m_2 |\vec{p}_1| c$$