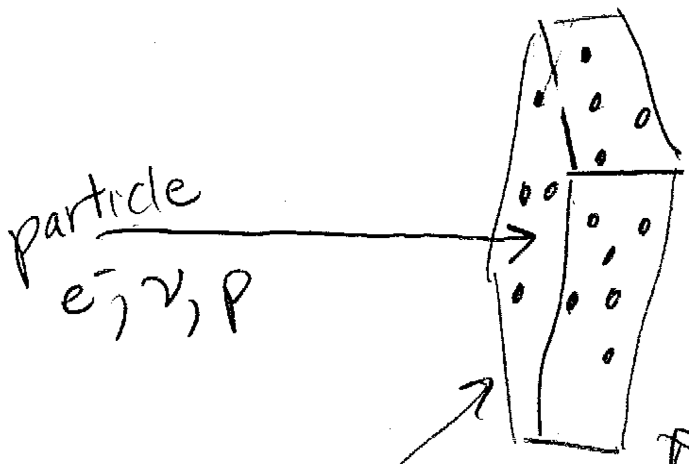
$$\Gamma \text{ "}\propto\text{" } |A_1|^2 + |A_2 + A_2'|^2 + \ldots$$

$$\propto \Gamma_1 + \Gamma_2$$

Feynman Calculus: how to go from diagrams to "partial rates"

Cross sections:

not just decay.. want to add energy. Concept: (very experimental)



scatterers $\rightarrow$ z $\leftarrow$ Probability the particle "does it"

Number density ρ N/volume

it $\rightarrow$ scatter into $d\theta\, d\phi$

$\rightarrow$ have reaction $pp \rightarrow ppp\bar{p}$ etc

Probability of n decays in non-zero & non-infinitesimal time interval T

n integrals

$$P(n) = \frac{(\Gamma T)^n}{n!} e^{-\Gamma T} = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!}$$

don't over count

note $$\sum_{n=0}^{\infty} P(n) = e^{-\Gamma T} \sum_{n=0}^{\infty} \frac{(\Gamma T)^n}{n!}$$

$$= 1$$

$$\langle n \rangle = \sum_{n=0}^{\infty} n P(n)$$

$$= \sum_{n=0}^{\infty} n \frac{(\Gamma T)^n}{n!} e^{-\Gamma T}$$

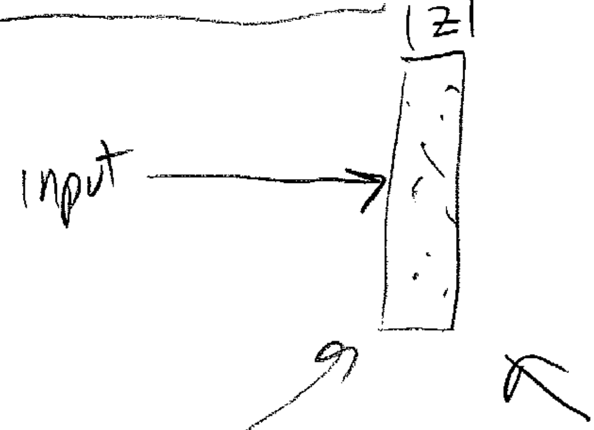
$$= \sum_{n=1}^{\infty} \frac{(\Gamma T)^n}{(n-1)!} e^{-\Gamma T}$$

$$= (\Gamma T) \underbrace{\sum_{n=1}^{\infty} \frac{(\Gamma T)^{n-1}}{(n-1)!} e^{-\Gamma T}}_{1}$$

$$\boxed{\langle n \rangle = \Gamma T \quad \text{mean \# of events}}$$

Two points:

Cross Section:



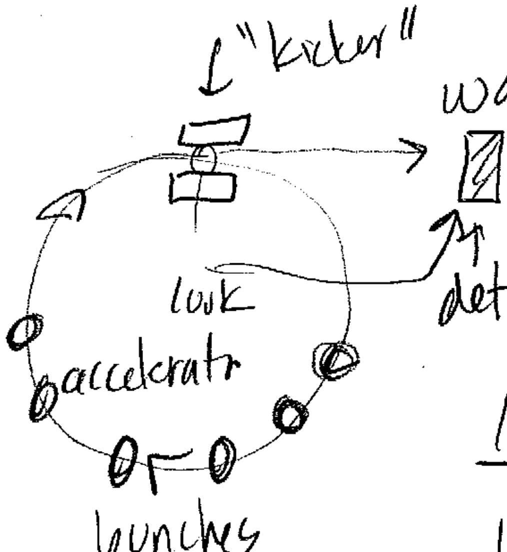
$$\boxed{\#\text{ interactions} \quad \langle n \rangle = \rho\, \sigma_{tot}\, z}$$

σ_{tot}

$\rho \; \frac{\text{scatterers}}{cm^3}$

$$P(0) = e^{-\langle n \rangle} = e^{-\rho \sigma_{tot} z}$$

"Practicle Point"



want 1 particle / bunch .. on average

detector ... 100% efficiency.

Measure: $P(0)$

want $\langle n \rangle = 1$

$$P(0) = e^{-1} \simeq 37\%$$

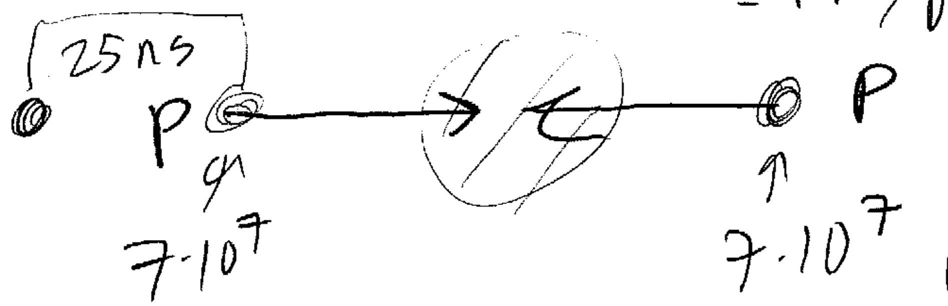
Then $P(1) = \frac{1}{1!} e^{-1} = 37\%$ $\quad P(2) = \frac{(1)^2}{2!} e^{-1} = 18\%$

$$P(3) = \frac{(1)^3}{3!} e^{-1} = 6.1\%$$

Where does this matter

LHC :

many interactions $\simeq 17$ / beam crossing



Interesting

$$\text{variance} = \sum_{n=0}^{\infty} (n - \langle n \rangle)^2 P(n)$$

$$= \Gamma T = \langle n \rangle \text{ (Homework)}$$

$$\boxed{\text{st. deviation} = \sqrt{\langle n \rangle}}$$ Poisson.

Very important.

prob of interaction $P \propto z, \rho_N$ true when

$$= \underbrace{\rho_N z}_{\text{dimensions?}} \cdot \sigma_i \ll 1 \quad \left(= \text{mean \# interactions}\right).$$

$\frac{\#}{cm^2}$

i refers to possible

Area $\perp$ to direction of motion.. relativistic invariant..

$\# / cm^2$

Total cross section characterizes "depletion" of intial beam --

$$\sigma_{tot} = \sum_i \sigma_i$$

Famous: $\boxed{\text{low energy neutrino, DM} \quad \sigma \sim 10^{-43}\ cm^2 / \text{nucleon}}$

thickness of water, 1 interaction

Water ρ: $1\ gm/cm^3$

Wt: $H_2O = 2 + 16 \simeq 18\ gm/mole$.

but: 18 nucleons/molecule...

$$\frac{\#\text{ nucleons}}{cm^3} = \rho_N \cong \frac{1}{18} \times 18 \cdot 6 \cdot 10^{23}$$

$$\simeq \underbrace{6 \cdot 10^{23} \text{ nucleons}/cm^3}_{\text{"true" meaning of Avogadro}}$$

want $6 \cdot 10^{23} \cdot z \cdot 10^{-43} = 1$

$$z = \frac{1}{6} \cdot 10^{20} \text{ cm}$$

$$\cong 1.7 \cdot 10^{19} \text{ cm}$$

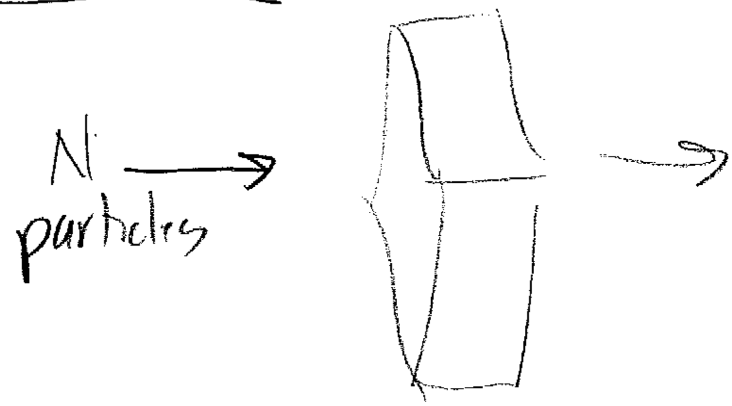
Light year: $3 \cdot 10^{10} \frac{cm}{s} \cdot 3 \cdot 10^{7}\ s/year$

$$\cong 10^{18} \text{ cm}$$

~ 20 light years of water to get 1 ν interaction

How do we detect? Start with 10^{20} or so ν!

Actually

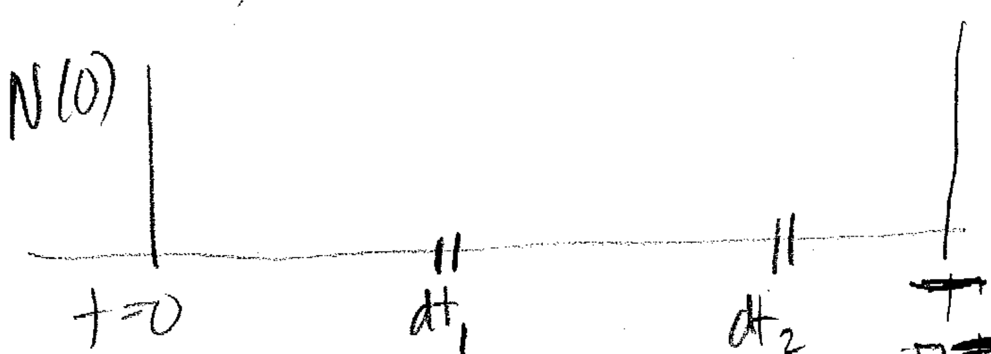


$$dN = -N\rho_N\sigma_{tot}\,dz$$

$$N(z) = N(0)\,e^{-\rho_N\sigma_{tot} z}$$

like $N(t) = N(0)\,e^{-\Gamma t} = N(0)\,e^{-t/\tau}$

Multiple interactions/decays in finite distance/time?



$N(T) = \#$ survive $= N(0)\,e^{-\Gamma T}$

$P_0(T) =$ prob of zero decays $= e^{-\Gamma T}$

$$P_1(T) = \int_0^T \underbrace{e^{-\Gamma T}}_{\text{no decays}} (\Gamma\,dt_1) = \Gamma T\,e^{-\Gamma T}$$

$$P_2(T) = \frac{1}{2!}\int_0^T\int_0^T e^{-\Gamma T}\,\Gamma dt_1\,\Gamma dt_2 = \frac{(\Gamma T)^2}{2!}e^{-\Gamma T}$$