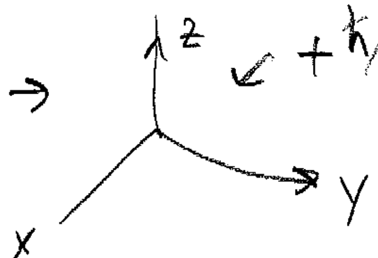
Spin Eigenstate
in θ, ϕ direction

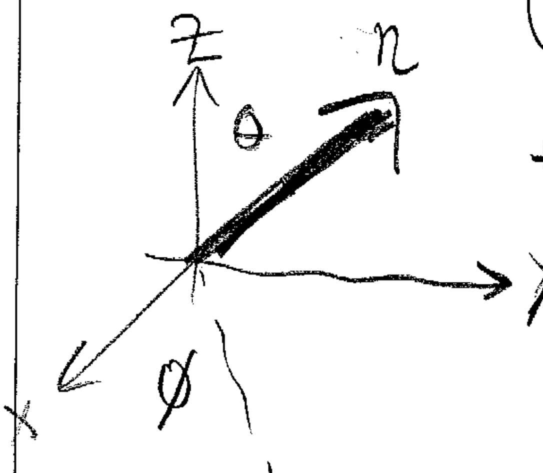
$$= \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2}e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

eigenstates in $\underline{z\text{-basis}}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow$ $+\hbar/2$ in z



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow -\hbar/2$ in z



$+\frac{\hbar}{2}$ along n direction is $\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\phi} \end{pmatrix}$

in z basis

$$\left.\begin{matrix} \cos^2\frac{\theta}{2} & \text{spin up along } z \\ \sin^2\frac{\theta}{2} & \text{spin down along } z \end{matrix}\right\} 1$$

subtle

$$\underset{\sim}{U}^{\dagger}(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} e^{-i\phi} \\ \sin\frac{\theta}{2} e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\underset{\sim}{U}(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} e^{-i\phi} \\ -\sin\frac{\theta}{2} e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\underset{\sim}{U}(\theta)^{\dagger}\underset{\sim}{U}(\theta) = \begin{pmatrix} \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} & \left(\cos\frac{\theta}{2}\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)e^{-i\phi} \\ \left(\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)e^{i\phi} & \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \leftarrow \text{"Unitary"}$$

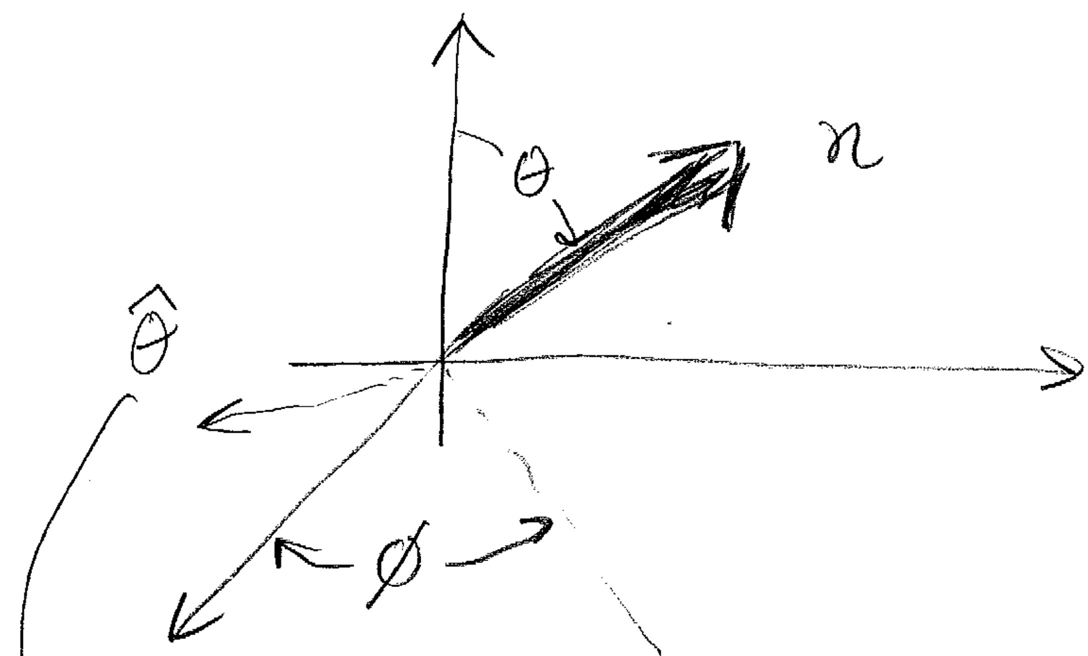
$\underset{\sim}{U}(\theta)$ above <u>are</u> a representation of $SU(2)$

<u>Note</u>:

$$\underset{\sim}{U}(\theta) = \cos\frac{\theta}{2}\cdot\underset{\sim}{1} - i\sin\frac{\theta}{2}\overbrace{\left(\sin\phi\,\underset{\sim}{\sigma}_x - \cos\phi\,\underset{\sim}{\sigma}_y\right)}^{\text{not so obvious}}$$

$$\sin\theta/2 \begin{pmatrix} 0 & -i\sin\phi + \cos\phi \\ -i\sin\phi - \cos\phi & 0 \end{pmatrix}$$

Define $\hat{\theta} = \hat{x}\sin\phi - \hat{y}\cos\phi$!



is $\perp$ to the ϕ direction!

$$\underset{\sim}{U}(\theta) = \cos\frac{\theta}{2}\,\underset{\sim}{\mathbb{1}} - i\sin\frac{\theta}{2}\left[\hat{\theta}\cdot\vec{\underset{\sim}{\sigma}}\right]$$

means $\underset{\sim}{\sigma}_x\sin\phi - \underset{\sim}{\sigma}_y\cos\phi$

$$e^{-i\frac{\alpha}{2}} = \cos\left(\frac{\alpha}{2}\right) - i\sin\left(\frac{\alpha}{2}\right)$$

$$e^{-i\frac{\theta}{2}\hat{\theta}\cdot\vec{\underset{\sim}{\sigma}}} = \cos\frac{\theta}{2}\,\underset{\sim}{\mathbb{1}} - i\sin\frac{\theta}{2}\left[\hat{\theta}\cdot\vec{\underset{\sim}{\sigma}}\right]$$

(Spin $\frac{1}{2}$) ... When $\underset{\sim}{H}$ has no preferred direction,

$$[\underset{\sim}{H}, \underset{\sim}{U}^{\dagger}(\theta)] = 0 \quad \forall \theta$$

<u>or</u> $[\underset{\sim}{H}, \underset{\sim}{\sigma}_x] = [\underset{\sim}{H}, \underset{\sim}{\sigma}_y] = [\underset{\sim}{H}, \underset{\sim}{\sigma}_z] = 0$

simultaneous eigenstates.

($u \leftrightarrows d$) Generalized Spin-$\frac{1}{2}$

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \underset{\sim}{U}_I^{\dagger}(\theta) \begin{pmatrix} u \\ d \end{pmatrix}$$

for isospin

Your definition of up & down

My definition

(Remember, electric charge neglected)

Sees same physics

$\therefore [\underset{\sim}{H}, \underset{\sim}{U}_I(\theta)] = 0$

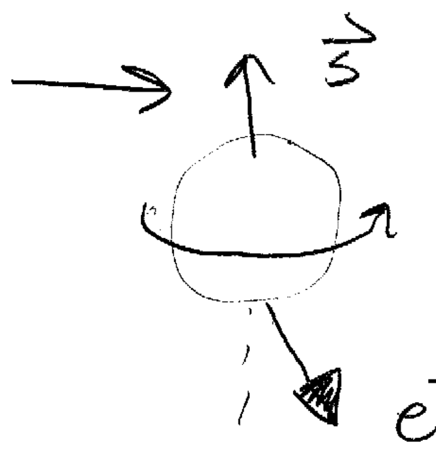
simultaneous eigenstates

Parity Violation

Means $[\underset{\sim}{H}_w, \underset{\sim}{P}] \neq 0$!

The experiment (C S Wu)

get all ^{60}Co lined up!

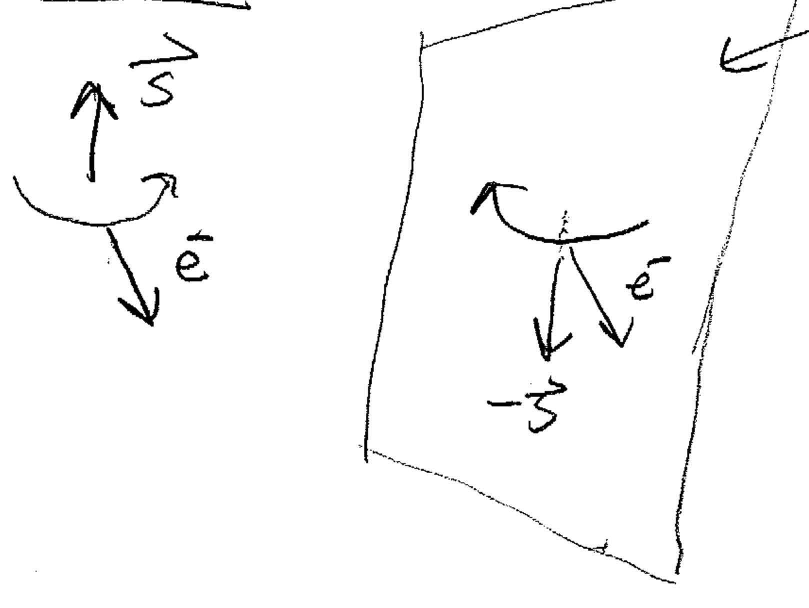


$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}_e$

measure that

come out preferentially opposite to ^{60}Co spin direction

Mirror



NOT SAME!