In either case..

$$\underbrace{\tfrac{1}{2} \otimes \tfrac{1}{2}} = 1 \oplus 0$$

$$\underbrace{2 \times 2}_{\text{4 states}} = \underbrace{3 + 1}_{\text{triplet + singlet}}$$

More general ? (No proof here)

j_1 j_2] combine ...

example (2 $\frac{3}{2}$)

Largest $J_{max} = j_1 + j_2$ $(3\frac{1}{2})$

Smallest $\underbrace{J_{min} = |j_1 - j_2|}_{\text{all in between too}}$ $(\frac{1}{2})$

$\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$

Count states.

$2 \rightarrow -2, -1, 0, 1, 2 \rightarrow 5$

$\frac{3}{2} \rightarrow$ $\times 4$

20 states.

$\frac{1}{2}$ $2 \times \frac{1}{2} + 1 = 2$

$1\frac{1}{2}$ 4

$2\frac{1}{2}$ 6

$3\frac{1}{2}$ 8 — 20 states !

Generally, how do you relate the two? → "Clebsch-Gordan"

One is always simple, "Max"

$J\ M$ remember $j_1 = 2$, $j_2 = 3/2$

$$\left|\tfrac{7}{2}\ \tfrac{7}{2}\right\rangle = \left|2\ \tfrac{3}{2}\right\rangle$$

$m_1 \quad m_2$

$$\left|\tfrac{7}{2}\ \tfrac{5}{2}\right\rangle = a\left|2\ \tfrac{1}{2}\right\rangle + b\left|1\ \tfrac{3}{2}\right\rangle$$

$a + b$ p 337 RPP!

$$= \sqrt{\tfrac{3}{7}}\left|2\ \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{4}{7}}\left|1\ \tfrac{3}{2}\right\rangle$$

$$\left|\tfrac{7}{2}\ \tfrac{3}{2}\right\rangle = \sqrt{\tfrac{1}{7}}\left|2\ -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{4}{7}}\left|1\ \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{7}}\left|0\ \tfrac{3}{2}\right\rangle$$

$$\text{Spin-}\tfrac{1}{2} \longleftrightarrow \underbrace{\text{Generic}}_{\text{iso spin}}\ 2\times 2$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

← • Diagonal
• e.v. along diagonal

$$\left.\begin{matrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \to 100\% \ +\hbar/2 \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to 100\% \ -\hbar/2 \end{matrix}\right\}$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$|\alpha|^2 \quad +\hbar/2$

$|\beta|^2 \quad -\hbar/2$

Isospin Resonances

$\lvert\pi^+\rangle$	$\lvert\pi^0\rangle$	$\lvert\pi^-\rangle$
$u\bar{d}$	$\frac{1}{\sqrt{2}}[u\bar{u} - d\bar{d}]$	
$I_3 = +1$	$I_3 = 0$	$I_3 = -1$
$I = 1$	$I = 1$	$I = 1$

Famous: (Fermi, 1950's)

vary $\vec{P}_\pi$

$\pi^+ p \to ?$

$\underbrace{\pi^- p}_{\text{strong scattering.}} \to ?$

uud

$p: I = \frac{1}{2} \quad I_3 = +\frac{1}{2}$
(?)

$n: I = \frac{1}{2}, I_3 = -\frac{1}{2}$

(not $I = \frac{3}{2}$, other 2 not observed)

$\pi^+ p \xrightarrow{\text{strong}} "\Delta^{++}"(1232) \qquad \sigma_+$

$\underbrace{I_3 = 1 \quad I_3 = \frac{1}{2}}$ $\quad I = \frac{3}{2}\ I_3 = +\frac{3}{2}$] one of 4!

$I_{3\,tot} = \frac{3}{2}$

$I_{tot} = \frac{3}{2}$

$I_{31}\ I_{32} \qquad I_{tot}\ I_{tot\,3}$

$\lvert 1\ \tfrac{1}{2}\rangle = \lvert \tfrac{3}{2}\ \tfrac{3}{2}\rangle$.

$\langle \tfrac{3}{2}\ \tfrac{3}{2} \lvert \underbrace{H_{strong}}_{\text{commutes}} \rvert 1\ \tfrac{1}{2}\rangle \propto \langle \tfrac{3}{2}\ \tfrac{3}{2} \vert 1\ \tfrac{1}{2}\rangle = 1$

$\overset{-1}{\pi^-}\ \overset{\frac{1}{2}}{p} \to \underbrace{\Delta^0(1232)}$

$I_{3\,tot} = -\frac{1}{2}$

$\lvert \tfrac{3}{2}\ -\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\lvert 0\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\lvert -1\ \tfrac{1}{2}\rangle$

$$\langle \tfrac{3}{2}\ -\tfrac{1}{2} | \mathcal{H}_{\text{strong}} | -1\ \tfrac{1}{2} \rangle$$

$$I_{tot}\ I_{3tot} \qquad I_{31}\ I_{32}$$

$$\propto \Big[\sqrt{\tfrac{2}{3}}\,\underbrace{\langle 0\ -\tfrac{1}{2}| + \sqrt{\tfrac{1}{3}}\,\langle -1\ \tfrac{1}{2}|\Big]\Big[|-1\ \tfrac{1}{2}\rangle}_{\text{orthogonal}}$$

$$\propto \sqrt{\tfrac{1}{3}}$$

Rate $\propto \frac{1}{3}$

$$\frac{\sigma_+}{\sigma_-} = \frac{1}{1/3} = 3$$

Look at page 134!

<u>Like Magic</u>