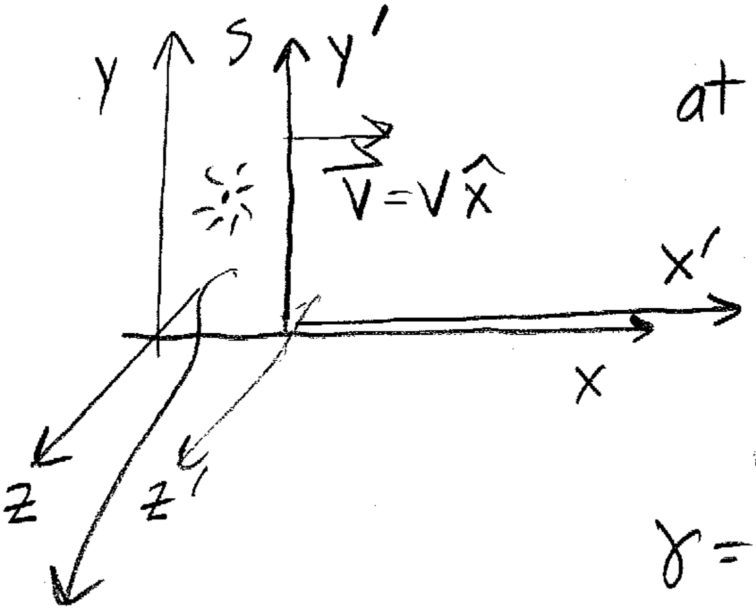
Relativistic Kinematics



at $t = t' = 0$, ORIGINS COINCIDENT

$$\beta \equiv \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

EVENT at x, y, z, t — not raise to power superscipt

<u>4 vector</u>: $x^0 \equiv ct \quad x^1 \equiv x \quad x^2 \equiv y \quad x^3 \equiv z$

same event $\rightarrow$ $x^{0\prime} = ct' \quad x^{1\prime} = x' \quad x^{2\prime} = y' \quad x^{3\prime} = z'$

"contravariant" 4-vector

LORENTZ

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma(ct - \beta x)$$

or

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\underset{\sim}{\Lambda} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For this L.T.

Λ^{μ}_{ν} : $\Lambda^{0}_{0} = \gamma$ $\Lambda^{1}_{0} = \Lambda^{0}_{1} = -\gamma\beta$

Cool Part (notationally):

$$x^{\mu\prime} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} \equiv \underbrace{\Lambda^{\mu}_{\nu} x^{\nu}}$$

repeated indices <u>imply</u> summation

check... one ↑ one ↓

"The Invariant"

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \quad \text{(prob 3.8)}$$

$$= \underbrace{(x^{0\prime})^2 - (x^{1\prime})^2 - (x^{2\prime})^2 - (x^{3\prime})^2}_{\text{4-dimensional "LENGTH"}}$$

4- dimensional "LENGTH"

$$g \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$ ← "The metric"

$g_{00} = 1$ $g_{11} = g_{22} = g_{33} = -1$

<u>covariant</u>: $x_\mu \equiv g_{\mu\nu} x^\nu$

or $x_0 = x^0 \quad x_1 = -x^1 \quad x_2 = -x^2 \quad x_3 = -x^3$

4-"dot" $\left[x^\mu x_\mu \equiv x^\mu x_\mu = x^\mu g_{\mu\nu} x^\nu = x_\nu x^\nu \right.$

$= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$

4-DOT PRODUCT

$a = (a^0, a^1, a^2, a^3)$

↑

no vector, no extra notation

$b \equiv (b^0, b^1, b^2, b^3)$

$a \cdot b$ or $ab \equiv a^0 b^0 - \vec{a} \cdot \vec{b}$] looks same in all frames!

$a \to p_A$ (E/momentum) particle a

$b \to p_B$ " particle b.

$a \cdot a = a^2 = (a^0)^2 - |\vec{a}|^2$

$a^2 > 0$ timelike

$a^2 < 0$ spacelike

$a^2 = 0$ lightlike

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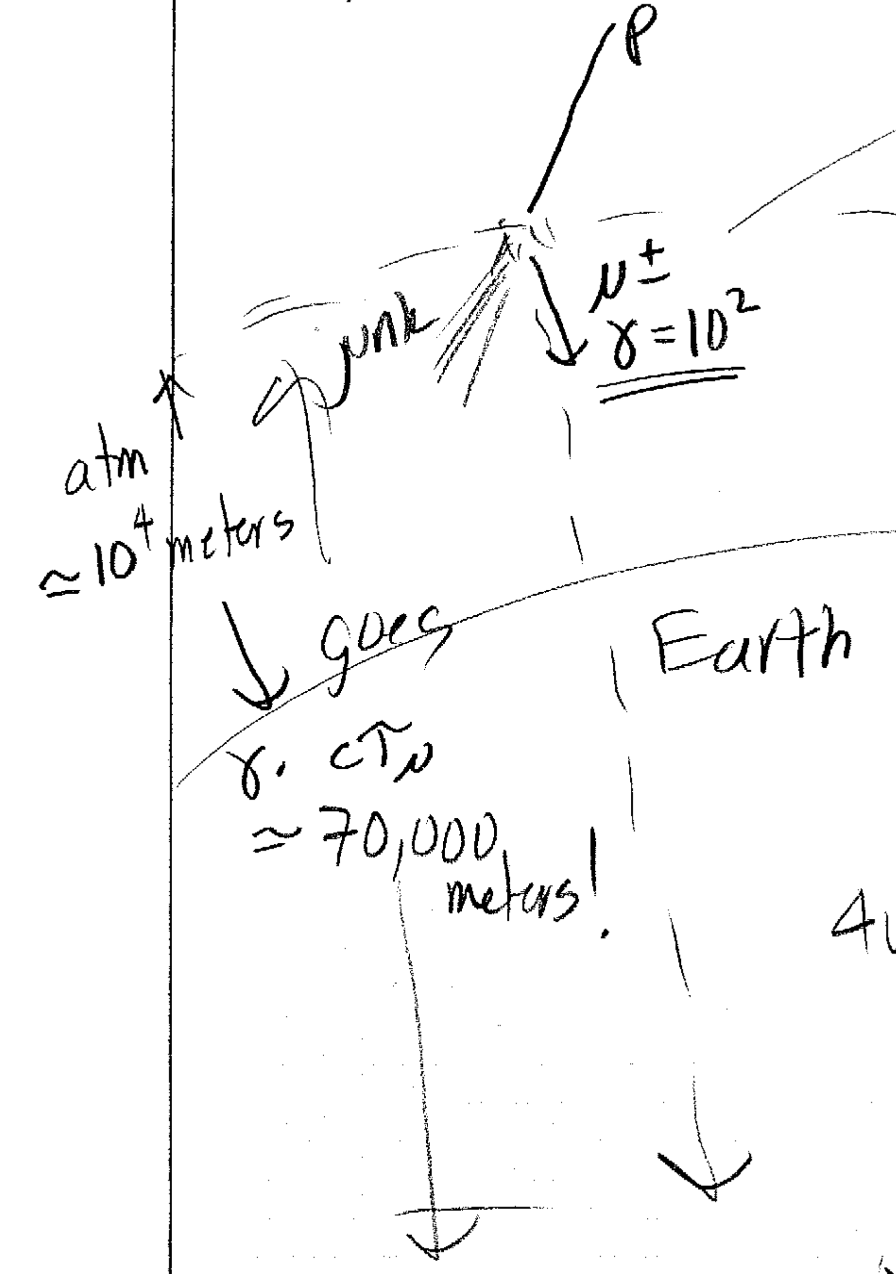
Proper Time



rest frame

$$P(\text{survive}) \propto e^{-\tau/\tau_\mu}$$

$\tau_\mu \simeq 2 \cdot 10^{-6}\ \mu s$

$\simeq 2 \cdot 10^{3}$ ns

$\simeq 2000$ ft

$\simeq 700$ m

4vector

$$p = m\eta = \left(\frac{E}{c}, \vec{p}\right)$$

$$t_{\text{earth}} = \gamma\tau \Leftarrow \text{rest frame}.$$

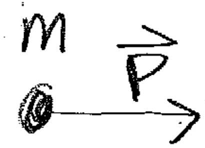
$$\boxed{\text{proper velocity}} \quad \vec{\eta} \equiv \frac{d\vec{x}}{d\tau} \begin{array}{l} \Leftarrow \text{earth frame} \\ \Leftarrow \mu \text{ rest frame} \end{array} = \gamma\vec{v}$$

$$\eta^0 \equiv \frac{d(c\,t_{earth})}{d\tau} = \gamma c$$

$$\eta^{0\,2} - |\vec{\eta}|^2 = \gamma^2 c^2 - \gamma^2 |\vec{v}|^2 = c^2 \frac{1}{1-\beta^2}\left(1 - \frac{|\vec{v}|^2}{c^2}\right) = c^2$$

What good is the 4-dot product?

Energy Momentum 4-vector



p or p^ν

$$E = \sqrt{(mc^2)^2 + (c|\vec{p}|)^2} \qquad = \left(\frac{E}{c}, \vec{p}\right)$$

$$p^2 = p \cdot p = p_\nu p^\nu$$

$$= \frac{E^2}{c^2} - |\vec{p}|^2 = m^2 c^2$$

Shorthand: $c = \underline{1}$!

$E, m, |\vec{p}|$ in MeV, GeV, TeV

$$E^2 - |\vec{p}|^2 = m^2$$

Most important .. "FRAME SHIFTING"

this collide ... what is ... the largest mass particle that can be produced in a collision? M

$$M \neq \frac{1}{c^2}\left(\overbrace{\sqrt{(m_A c^2)^2+(c|\vec{p}_A|)^2}}^{E_A} + \overbrace{\sqrt{(m_B c^2)^2+(c|\vec{p}\,|)^2}}^{E_B}\right)$$

Because energy wasted on translating the center of mass!

<u>Imagine</u> a Lorentz Frame where..... (center of 'mass' frame)

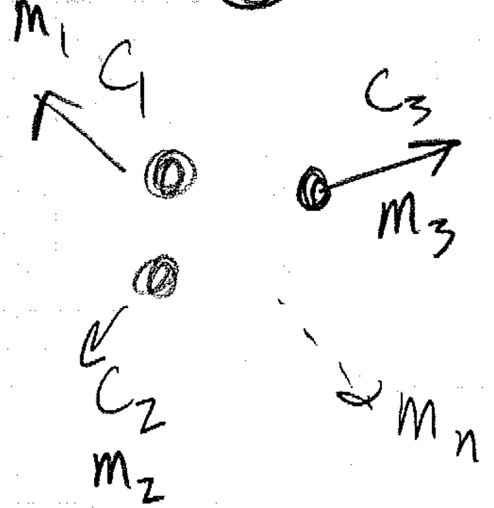
before

$$p_1^* = \left(\frac{E_1^*}{c}, \vec{p}^*\right) \qquad p_2^* = \left(\frac{E_2^*}{c}, -\vec{p}^*\right)$$

4-vector

$$M = \frac{1}{c^2}(E_1^* + E_2^*)$$

during



after

$$M \geq \sum_{i=1}^{n} m_i$$

Note: $p_1^* + p_2^* = \frac{1}{c}(E_1^* + E_2^*, \vec{0})$

IN CM FRAME

$$\underbrace{(p_1^* + p_2^*)^2}_{\text{4-vector}} = \frac{1}{c^2}(E_1^* + E_2^*)^2 - \vec{0}^2$$

4-vector $= (Mc)^2$

"Frame Independence"

$$= (p_{A\,Lab} + p_{B\,Lab})^2 \quad !$$

$$= \left[\left(\frac{E_A}{c} + \frac{E_B}{c} \right), \vec{p}_A + \vec{p}_B \right]^2$$

$$M^2c^2 = \frac{1}{c^2}(E_A + E_B)^2 - (\vec{p}_A + \vec{p}_B)^2$$

this accounts for motion of CM

Very Famous (1955)

$$p + p \rightarrow \underbrace{p + p + p + \bar{p}}$$

momentum $\vec{p}$; rest

$$Mc^2 > \sum m_i c^2$$

$$= 4 m_p c^2$$

$$= 4 \cdot 938.3 = 3753.2 \text{ MeV}$$

$$\underset{\text{MeV}}{(3753.2)^2} \leq (E + \underbrace{m_p c^2}_{\substack{\text{rest}\\\text{energy}\\\text{of 1 proton}}})^2 - c(\vec{p} + \vec{0})^2$$

total energy

$$\leq E^2 + 2Em_pc^2 + (m_pc^2)^2 - c^2|\vec{p}|^2$$

$$\leq 2m_pc^2(E + m_pc^2)$$

$$E \geq \frac{(3753.2)^2}{2m_pc^2} - m_pc^2$$

$$\geq \frac{16}{2}\frac{m_p^2c^4}{m_pc^2} - m_pc^2$$

$$E \geq 7m_pc^2 = 6568 \text{ MeV}$$

$$cp \geq \sqrt{E^2 - (m_pc^2)^2}$$

$$\geq \sqrt{(49-1)(m_pc^2)^2} = 6500 \text{ MeV}$$

$$T = E - m_pc^2 = 6m_pc^2 = 5629 \text{ MeV}$$

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<u>Decay</u>: $A \to B + C$

$$\left.\begin{array}{l} m_A \to m_B + m_C \\ m_A > m_B + m_C \end{array}\right\} \text{ want } E_B, E_C$$

Look at rest frame of A...

$$B \xleftarrow{\;-\vec{p}^*\;} \overset{A}{\bullet} \xrightarrow{\;\vec{p}^*\;} C$$

$\Leftarrow$ in rest frame of A

$$p_A = p_B + p_C \Leftarrow \text{4-momenta}$$

<u>Trick #1</u>

(CHOOSE!)

$$p_A \cdot p_B = \text{look at rest frame}$$
$$= \left(m_A c, \vec{0}\right) \cdot \left(\frac{E_B}{c}, -\vec{p}^*\right)$$
$$= m_A \cdot E_B$$

<u>Trick #2</u>

$$(p_A - p_B) = p_C$$

$\boxed{\text{SQUARE BOTH SIDES}}$

$$(p_A - p_B)^2 = p_C^2 = (m_C c)^2$$

$$p_A^2 - 2 p_A \cdot p_B + p_B^2 = (m_C c)^2$$

$$(m_A c)^2 - 2 m_A E_B + (m_B c)^2 = (m_C c)^2$$

$$\boxed{E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2 m_A} c^2}$$

$$\boxed{E_C = \frac{m_A^2 - m_B^2 + m_C^2}{2 m_A} c^2}$$

<u>note</u>! $|\vec{p}^*| = \sqrt{\left(\frac{E_B}{c}\right)^2 - (m_B c)^2}$

$$= \sqrt{\left(\frac{E_C}{c}\right)^2 - (m_C c)^2}$$

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