Physics 125 Problem Set 6

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1. The derivation of the neutrino oscillation formula, 11.8 on page 392 of the text, assumed that the momenta of the neutrino eigenstates produced in the sun were equal. Actually, the *energies* are the same, and it is the momenta that are unequal... but you get the same answer, to lowest order in $\Delta m^2 = m_2^2 - m_1^2$. Let's explore the situation in this problem, for a representative solar process, which is electron capture by ^{7}Be.

   (a) In the process of electron capture by ^{7}Be:

   $$e^- +^7 \text{Be} \rightarrow^7 \text{Li} + \nu_e$$

   the *atomic* mass of the ^{7}Be, which includes the masses of all the electrons, is $M_A = 7.0169292\,\text{amu}$ (atomic mass unit); it is very useful to know that the mc^2 of one amu is $931.494028\,\text{MeV}$, and to work in the rest frame of the ^{7}Be. The atomic mass of ^{7}Li is $M_B = 7.016004\,\text{amu}$. For this part, take $M_C = 0$ (in the equations of problem 3.19 on page 112 of the text); that is, take the neutrino to be massless, and evaluate symbolically and numerically the energy of zero mass neutrino, as seen in the ^{7}Be rest frame, $E_0^\star$.

   (b) Now imagine M_C to be either m_1 (the mass of one neutrino eigenstate) or m_2 (the mass of another); what will really matter is *differences* between the case of m_1 and m_2. Assume, as recently proven by KAMLAND, that $\Delta m^2 = m_2^2 - m_1^2 = 8 \times 10^{-5}\,\text{eV}^2/\text{c}^4$, and be really careful to use consistent units... eV or MeV. The easiest difference to evaluate is the *fractional* difference in neutrino energy, between the situation when the final state is m_2 and that when the final state is m_1, relative to the case where the final neutrino energy is zero. That is, evaluate (symbolically and numerically)

   $$\delta_E \equiv \frac{E_2^\star - E_1^\star}{E_0^\star},$$

   where the $\star$ means in the ^{7}Be rest frame and the subscript 0 means for a zero-mass neutrino, and 1 and 2 are for neutrinos of mass m_1 and m_2. The δ_E you compute will be a very small number!

   (c) Repeat the previous portion for the center of mass momentum, $p^\star$. Here, expand to lowest order in Δm^2... there are definitely higher order terms that will have to be dropped, unlike in the case of the energies. You should still get a tiny number for δ_p, but, it will be a whole lot bigger than δ_E. Comment on the portion of the formula that causes δ_p to be much bigger.

   (d) In the text, propagation in time is carried out by the factor $e^{-iEt/\hbar}$. A more careful analysis would have used $e^{ip^\star z/\hbar - iE^\star t/\hbar}$, why is that (think... plane wave! not much more complicated than that). So, if you use the proper propagator, and make reasonable approximations, do you recover Equation 11.8 on page 392? Show it.

2. Griffiths 4.8

3. Griffiths 4.17
4. Griffiths 4.18
5. Griffiths 4.19
6. Griffiths 4.20
7. Griffiths 4.21