# Physics 115C Midterm

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Tuesday, October 29, 2002

Open book: you can use the textbook during the test. You can also use a calculator. Generally, express answers in symbolic form. Give numerical form when specified.
Dimensionless radial Schrödinger equation:

$$\left\{ -\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{\rho^2} + \mathrm{sgn}[V_0]\rho^s \right\} U_{\epsilon\ell}(\rho) = \epsilon U_{\epsilon\ell}(\rho)$$

where the potential $V(r) = V_0 r^s$, the dimensionless radius $\rho$ is related to the radius $r$ by $r = a\rho$ with $a = (\hbar^2/2\mu|V_0|)^{1/(s+2)}$ and the dimensionless energy $\epsilon = E/E_0$ with $E_0 = |V_0|(\hbar^2/2\mu|V_0|)^{s/(s+2)}$. The full wave-function is $\psi_{\epsilon\ell m} = R_{\epsilon\ell}(\rho)Y_\ell^m(\theta,\phi)$, and $U_{\epsilon\ell}(\rho) = \rho R_{\epsilon\ell}(\rho)$. The sgn function gives: $\mathrm{sgn}V_0 = +1$ when $V_0 > 0$, $-1$ when $V_0 < 0$.

To invert a 2 by 2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

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1. (30 pts) A particle is described by the wave function

$$\psi_E(r,\theta,\phi) = Ae^{-\gamma r^n} \ (A, \gamma = \text{real constants}, n = \text{non-zero integer})$$

   (a) (10 pts) What is the angular momentum content of the state?

   (b) (20 pts) This form of wave function can solve Schrödinger's equation for all $r$ for certain power law potentials with form $V(r) \propto r^s$. Find all the values of $s$ for which $\psi_E$ is a solution to Schrödinger's equation. Make a brief table showing those values of $s$ and the corresponding values of $n$.

2. (15 pts) Find the ratio of the energy of the ground state for the potential $V(r) = \gamma r$, $E_1$, to the energy of the ground state for the potential $\overline{V}(r) = 2\gamma r$, $\overline{E}_1$. That is, find $E_1/\overline{E}_1$. The constant $\gamma$ is a positive real number, and assume that the (reduced) mass of the bound particle (system) is the same for both potentials.

3. (30 pts) In this problem, obtain numerical answers, in terms of rational numbers and square roots like $\sqrt{2}$ and $\sqrt{3}$. An electron at rest is described by the spinor:

$$\begin{pmatrix} -\frac{1}{2}e^{-i\pi/8} \\ \frac{\sqrt{3}}{2}e^{i\pi/8} \end{pmatrix}$$

   (a) (15 pts) What are the expectation values of the operators $s_x$, $s_y$, and $s_z$?

   (b) (15 pts) The spinor is an eigenstate, with eigenvalue $+\hbar/2$, of a particular matrix. Find that matrix.

4. (25 pts) Let $\vec{A}$ be a real 3-vector, and $A_0$ be a simple real number. Find the four coefficients $m_\beta$ in the relationship

$$\frac{1}{A_0\mathbf{1} \ + \ \vec{A}\cdot\vec{\sigma}} = \sum_{\beta=0}^{3} m_\beta \sigma_\beta$$

   where the 4 $\sigma_\beta$ are the 2 by 2 identity matrix $\mathbf{1}$ (for $\beta = 0$) and the three Pauli matrices $\sigma_\beta$ (for $\beta = 1,2,3$). Feel free to start out by explicitly writing out the 2 by 2 matrix image for the denominator of the left hand side.

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