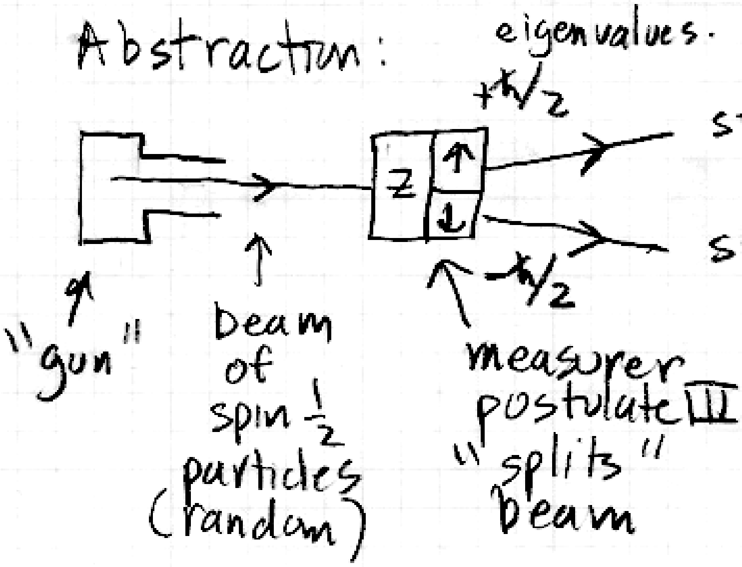
Stern Gerlach Apparatus (Postulate III p.116)

Abstraction:



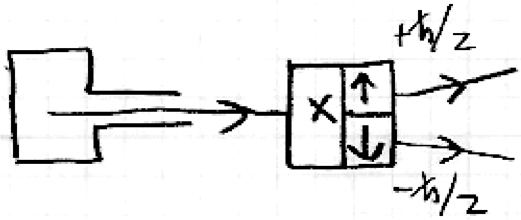
state $\begin{pmatrix}1\\0\end{pmatrix}$ 50%

state $\begin{pmatrix}0\\1\end{pmatrix}$ 50%

in $\underset{\sim}{S}_z$ eigenbasis

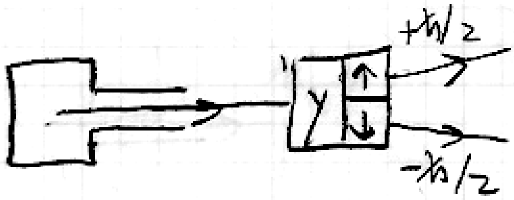
assuming gun is random.

Measurer: axis could be $\hat{z}$; $\hat{x}$; $\hat{y}$ or $\hat{n} \to (\theta, \phi)$



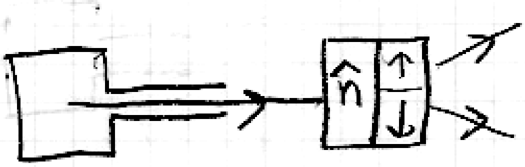
$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$



$\frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\pi/4}\\e^{+i\pi/4}\end{pmatrix}$

$\frac{1}{\sqrt{2}}\begin{pmatrix}-e^{-i\pi/4}\\e^{i\pi/4}\end{pmatrix}$



$\begin{pmatrix}e^{-i\phi/2}\cos\theta/2\\e^{i\phi/2}\sin\theta/2\end{pmatrix}$

$\begin{pmatrix}-e^{-i\phi/2}\sin\theta/2\\e^{i\phi/2}\cos\theta/2\end{pmatrix}$

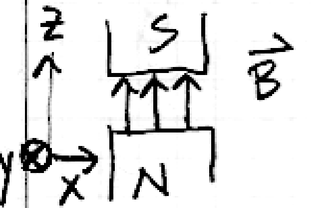
Under the hood of the measurer:

1) use a $\vec{B}$ field

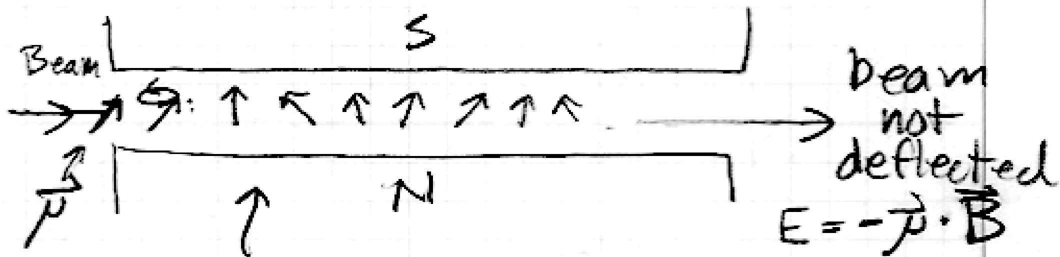
2) simplest: $\vec{B}$ uniform $\Rightarrow$ spin <u>precesses</u>

$\Rightarrow$ spin eigenvalues <u>not</u> disentangled.

Uniform field:



send beam in along y



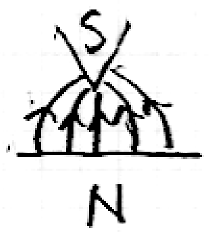
spin will precess as beam penetrates (distance $\propto$ time)

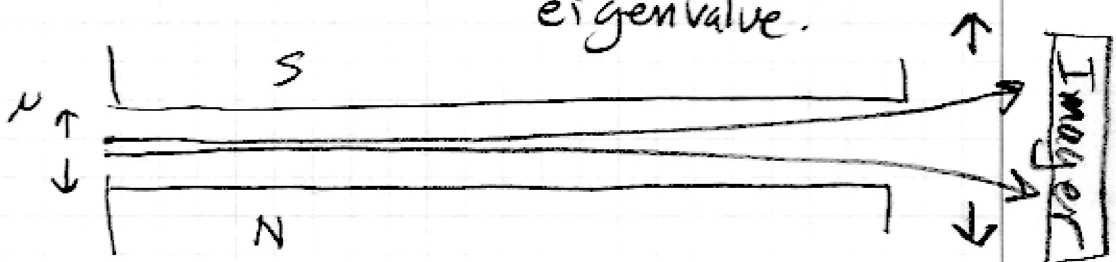
3) to get deflection, give $E = -\vec{\mu} \cdot \vec{B}$ a spatial gradient, $\parallel$ to $\vec{B}$

→ want $\frac{\partial E}{\partial z} \neq 0$, creates <u>Force</u>

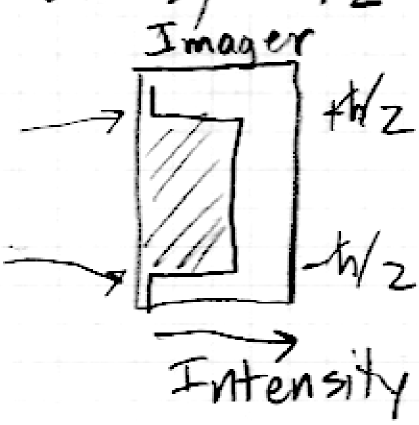
$$\propto \mu_z \frac{\partial B_z}{\partial z} \propto s_z \frac{\partial B_z}{\partial z}$$

eigenvalue.

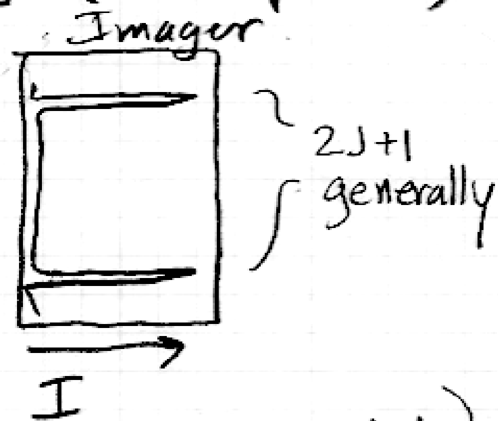




4) classically, s_z can take a continuum of values, $-\hbar/2$ to $+\hbar/2$ (for spin ½)



Classical



Quantum (!!!)

5) Experiment doesn't work for a beam of electrons... interplay of Lorentz force and $\vec{\nabla}\cdot\vec{B} = 0 = \frac{\partial B_z}{\partial z} + \overset{0}{\cancel{\frac{\partial B_y}{\partial y}}} + \frac{\partial B_x}{\partial x} = 0$

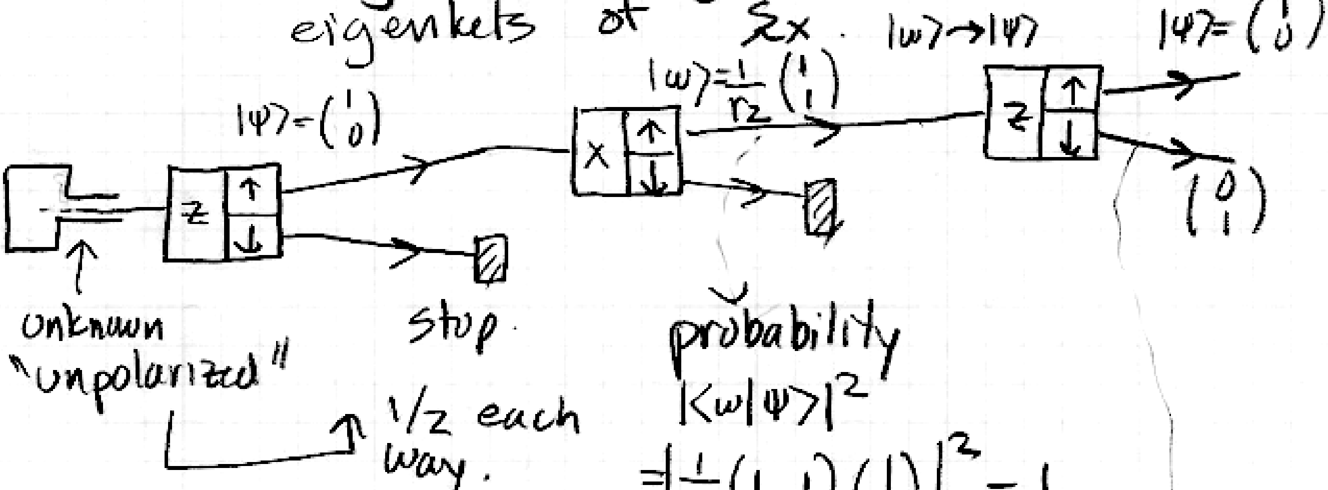
$B_x \neq 0$ everywhere; wave packet spreads to region of $B_x \neq 0$; $\hat{y}\times\hat{x} = -\hat{z}$, so beam gets blurred in z direction... washes out spin separation.

6) so use H or Silver. No net charge; no Lorentz force. Nucleus keeps electron "on a leash"!

Sequential Stern Gerlachs

Recall.. $[\underset{\sim}{S}_z, \underset{\sim}{S}_x] \neq 0$

... in general, eigenkets of $\underset{\sim}{S}_z$ are <u>not</u> eigenkets of $\underset{\sim}{S}_x$.



probability $|\langle\omega|\psi\rangle|^2$

$= \left|\frac{1}{\sqrt{2}}(1\ \ 1)\begin{pmatrix}1\\0\end{pmatrix}\right|^2 = \frac{1}{2}$

"up" probability $|\langle \overset{\text{new}}{\omega} | \overset{\text{new}}{\psi}\rangle|^2 = \left|(1\ \ 0)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \frac{1}{2}$

"down" probability $|\langle\omega|\psi\rangle|^2 = \left|(0\ \ 1)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \frac{1}{2}$

cumulative: $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$

$$|\omega_1\rangle = \begin{pmatrix} e^{-i\phi/2}\cos\theta/2 \\ e^{i\phi/2}\sin\theta/2 \end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\omega_2\rangle = \begin{pmatrix} -e^{-i\phi/2}\sin\theta/2 \\ e^{i\phi/2}\cos\theta/2 \end{pmatrix}$$

$$P_1 = |\langle\omega_1|\psi\rangle|^2 = \left| \left(e^{i\phi/2}\cos\frac{\theta}{2} \;\; e^{-i\phi/2}\sin\frac{\theta}{2} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \cos^2\frac{\theta}{2}$$

$$P_2 = |\langle\omega_2|\psi\rangle|^2 = \left| \left(-e^{i\phi/2}\sin\frac{\theta}{2} \;\; e^{-i\phi/2}\cos\frac{\theta}{2} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \sin^2\frac{\theta}{2}$$