Physics 115B Ninth Problem Set

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due 5pm Friday, June 6, 2003
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1. Exercise 12.4.3 page 320 of your text.
2. Exercise 12.5.2 page 329 of your text.
3. Exercise 12.5.3 page 329 of your text.
4. In this problem, work out the normalization of $Y_l^l(\theta,\phi)$ in Eq.(12.5.32) on page 335. Assume that the ϕ dependence of the function contributes the factor of $1/\sqrt{2\pi}$, and that the factor of $(-1)^l$ is merely the result of a convention. So, the task is to derive the factor:

$$\left[\frac{(2l+1)!}{2}\right]^{1/2}\frac{1}{2^l l!},$$

by recursion, through consideration of:

$$I_l = \int_0^\pi \sin^{2l+1}\theta d\theta.$$

(a) Start with the evaluation of:

$$I_0 = \int_0^\pi \sin\theta d\theta.$$

(b) Then use integration by parts to show:

$$I_l = \frac{2l}{2l+1} I_{l-1}.$$

(c) Conclude that:

$$\begin{aligned} I_l &= \frac{(2l)\times(2l-2)\times(2l-4)\times\ldots\times 6\times 4\times 2}{(2l+1)\times(2l-1)\times(2l-3)\times\ldots\times 5\times 3\times 1}\times 2 \\ &= \frac{[2^l l!]^2}{(2l+1)!}\times 2, \end{aligned}$$

...Over

and thus arrive at the normalization factor desired.
As a comment, sometimes the *double factorial* notation is used to simplify expressions such as that for I_l; the double factorial is: $n!! = n \times (n-2) \times (n-4) \times ... \times (2 \text{ or } 1)$. With this notation:

$$I_l = \frac{(2l)!!}{(2l+1)!!} \times 2$$

(d) Evaluate, with proper normalization, $Y_4^4(\theta, \phi)$ and $Y_4^3(\theta, \phi)$.

5. Exercise 12.5.13 page 338 of your text.