so, $c = i\gamma$

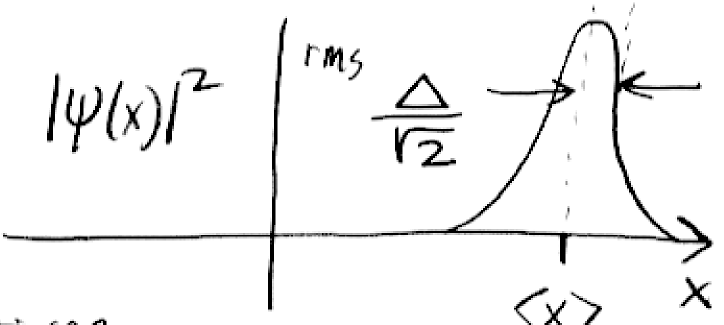
minimum uncertainty packet is:

$$\boxed{\begin{array}{l} \psi(x) = A e^{\frac{i\langle p\rangle x}{\hbar}} e^{\frac{-\gamma x'^2}{2\hbar}} \qquad x' = x - \langle x\rangle \\ \text{for } \psi(x) \to 0 \text{ as } x \to \pm\infty, \ \gamma > 0 \end{array}}$$

refer to p. 135 of the text;

$$\frac{\gamma}{\hbar} = \frac{1}{\Delta^2} \quad \text{or} \quad \gamma = \frac{\hbar}{\Delta^2}, \quad \Delta^2 = \frac{\hbar}{\gamma}$$

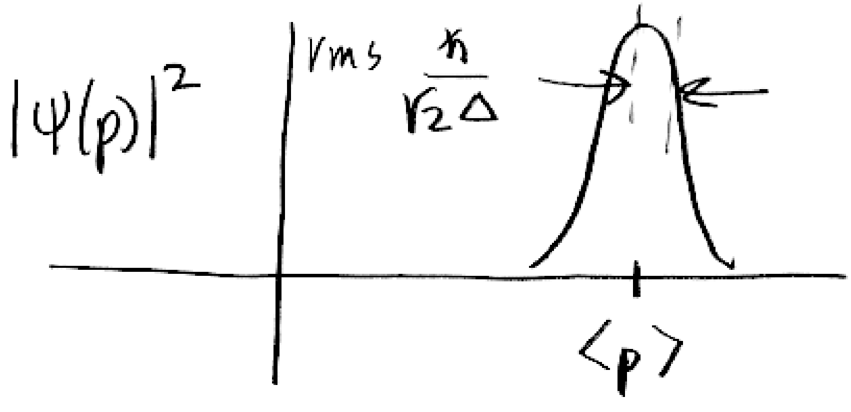
$$A = \frac{1}{(\pi\Delta^2)^{1/4}}$$



Δ quantifies the <u>width</u> of the distribution in space... Variance is $\Delta^2/2$.

$$\psi(p) = \frac{1}{(2\pi\hbar)^{1/2}} \int_{-\infty}^{\infty} dx \, e^{-ipx/\hbar} \, \psi(p) \qquad \text{(p. 137)}$$

$$= \left(\frac{\Delta^2}{\pi\hbar^2}\right)^{1/4} e^{\frac{-i(p-\langle p\rangle)\langle x\rangle}{\hbar}} e^{-(p-\langle p\rangle)^2\Delta^2/2\hbar^2}$$



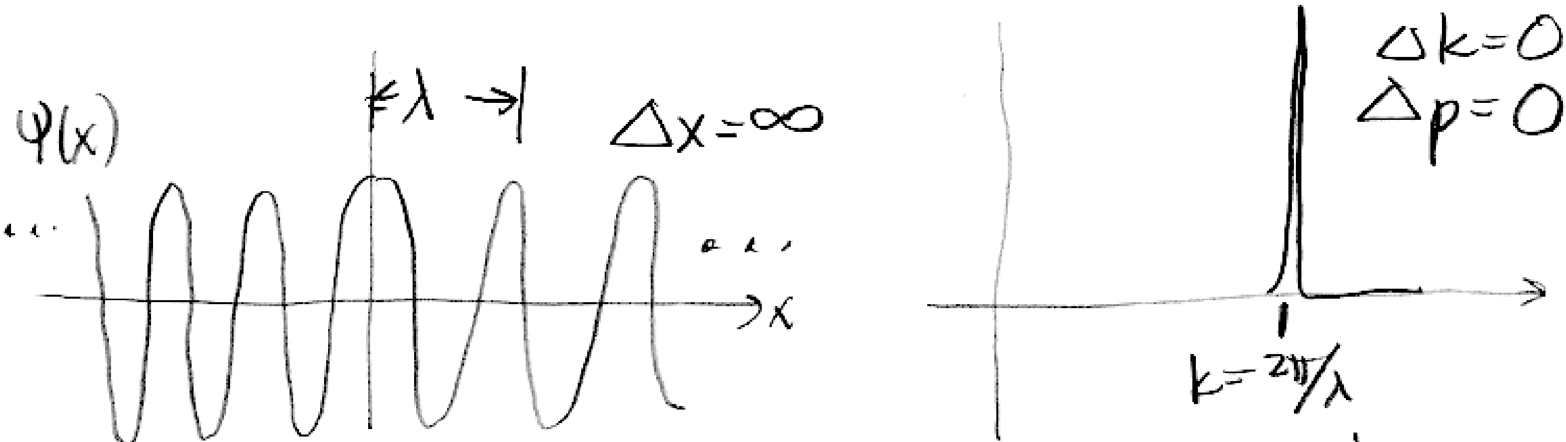
rms x × rms p

$$\frac{\Delta}{\sqrt{2}} \times \frac{\hbar}{\sqrt{2}\Delta} = \frac{\hbar}{2}$$

A more qualitative view

uncertainty.xls = Excel Spreadsheet
available on course page

idea:



one wavelength → δ-function in k-space / p-space

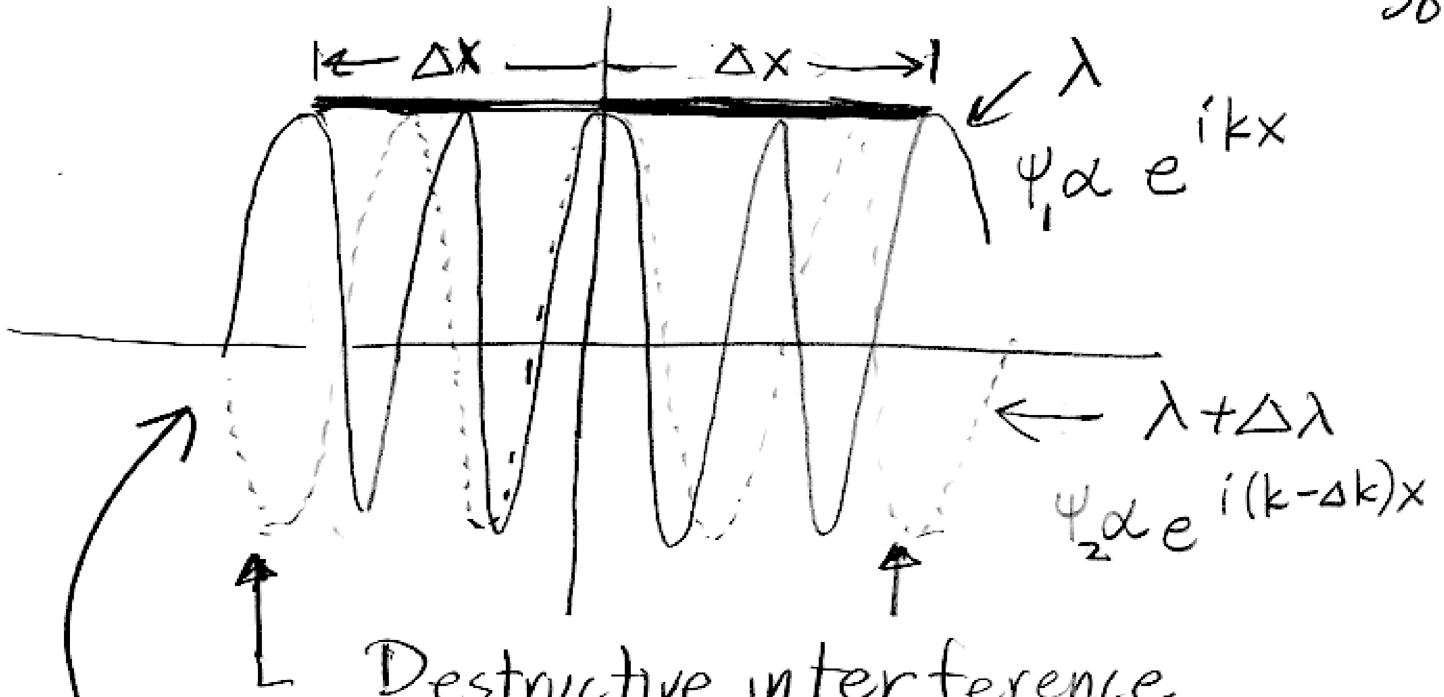
$$\psi \propto \cos\left(\frac{2\pi x}{\lambda}\right) = \mathrm{Re}\left(e^{\frac{2\pi i x}{\lambda}}\right) = \mathrm{Re}\left(e^{ikx}\right) = \mathrm{Re}\left(e^{\frac{ipx}{\hbar}}\right)$$

wavenumber $k = \frac{2\pi}{\lambda}$ $p = \hbar k$

---

now, say you want to attempt to localize ψ in some region $2\Delta x$ wide. Exploit the tool of superposition... blend in a wave with a slightly different wavelength $\lambda + \Delta\lambda$ and wavenumber $k - \Delta k$. How do you choose $\Delta\lambda$ or Δk?

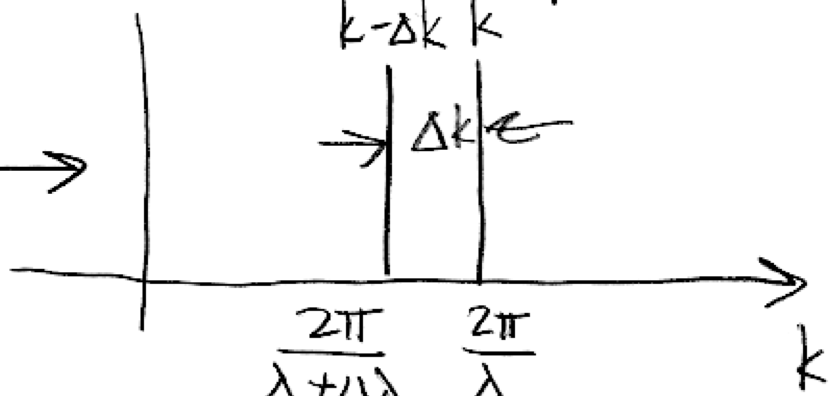
(recall, $\frac{2\pi}{\lambda + \Delta\lambda} \equiv k - \Delta k$)



Destructive interference at $\pm\Delta x$: $\psi_1 + \psi_2$ vanishes there.

$\psi_1 + \psi_2$ more localized in x-space

$\underbrace{\psi_1 + \psi_2}$ in k-space



$\psi_1 + \psi_2$ <u>less</u> localized in k-space.

want ψ_2 to have one fewer wavelength in the range $2\Delta x$, to get destructive interference

$$\frac{2\Delta x}{\lambda + \Delta\lambda} = \frac{2\Delta x}{\lambda} - 1 \qquad \text{(Key Point)}$$

or
$$\frac{2\pi}{\lambda + \Delta\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \cdot \Delta x - \pi$$

$$k - \Delta k \equiv \frac{2\pi}{\lambda + \Delta\lambda} \qquad k = \frac{2\pi}{\lambda}$$

$$(\cancel{k} - \Delta k) \cdot \Delta x = \cancel{k \Delta x} - \pi$$

$$\Delta k \, \Delta x = \pi$$

$$\text{or} \quad \underbrace{\hbar \Delta k}_{\Delta p} \, \Delta x = \hbar \pi$$

$$\boxed{\Delta p \, \Delta x = \hbar \pi}$$

- could have stuffed more than 1 wavelength in: 1, 3, 5, ...

  so $\Delta p \, \Delta x \geq \hbar \pi$
- could have made wavelength <u>smaller</u> rather than larger.
- Why π and not $\frac{1}{2}$? This approximation too simple. But it gets the point
- Δp is independent of $p = \frac{2\pi\hbar}{\lambda}$
- Fill in wavelengths <u>between</u> $\frac{2\pi}{\lambda}$ and $\frac{2\pi}{\lambda \pm \Delta\lambda}$: that suppresses peaks way outside $\pm \Delta x$ (see spreadsheet)

Uncertainty + Bound States p.341

The uncertainty principle (or, that particles are waves) keeps the world from collapsing.

$$\underset{\sim}{H} = \underbrace{\frac{p_x^2 + p_y^2 + p_z^2}{2m}}_{\substack{\text{kinetic energy} \\ \text{of } m = \text{electron mass} \\ \text{assuming proton } \infty \text{ mass}}} - \underbrace{\frac{e^2}{(\underset{\sim}{x}^2 + \underset{\sim}{y}^2 + \underset{\sim}{z}^2)^{1/2}}}_{\substack{\text{coulomb} \\ \text{attraction;} \\ \underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z} \text{ are} \\ \text{cordinate of } e^- \\ \text{w/r to proton.}}}$$

$$\langle \underset{\sim}{H} \rangle = \frac{\langle p_x^2 \rangle + \langle p_y^2 \rangle + \langle p_z^2 \rangle}{2m} - e^2 \left\langle \frac{1}{(\underset{\sim}{x}^2 + \underset{\sim}{y}^2 + \underset{\sim}{z}^2)^{1/2}} \right\rangle$$

seek to <u>minimize</u> $\langle \underset{\sim}{H} \rangle$; that will be the ground state. We'll find that the closer you squish the electron into the proton, thereby reducing the potential energy contribution, the ensuing localization pushes the kinetic energy up (via uncertainty principle). Lowest energy is result of a compromise between the two terms.