Physics 115B Final Exam

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Open Book: textbook, problems sets, solutions, and lecture notes allowed. For full credit, show your work and make your reasoning clear to the graders. Useful equations and information appears at the end of the test.

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1. (15 pts) Imagine a fictitious world in which the single-particle Hilbert space is two-dimensional. Let us denote the basis vectors by $|+\rangle$ and $|-\rangle$. Let

$$\sigma_1^{(1)} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2^{(2)} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

be operators in the space of particle #1 and the space of particle #2, respectively; the order of the states used to represent the operators is $|+\rangle$ and then $|-\rangle$. Find the 4 by 4 matrix that represents the operator $\mathbf{\Omega}$:

$$\mathbf{\Omega} = (\sigma_1\sigma_2)^{(1)\otimes(2)} = \sigma_1^{(1)} \otimes \sigma_2^{(2)};$$

use the order of the 4 product states as $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$.

2. (15 pts) A simple harmonic oscillator is in the state $|\psi\rangle$:

$$|\psi\rangle = \sqrt{\frac{1}{5}}|0\rangle - \sqrt{\frac{1}{5}}|11\rangle + \sqrt{\frac{3}{10}}|65\rangle - \sqrt{\frac{3}{10}}|98\rangle,$$

where $|n\rangle$ is an energy eigenket with energy eigenvalue $(n + \frac{1}{2})\hbar\omega$. The parity (with eigenvalue π) of $|\psi\rangle$ is measured. Evaluate the probabilities $P(\pi = +1)$ and $P(\pi = -1)$.

3. (25 pts) You evaluated the matrix representations of $\mathbf{J}_x = \mathbf{J}_1$, $\mathbf{J}_y = \mathbf{J}_2$, and $\mathbf{J}_z = \mathbf{J}_3$ for $j = 1$ (where $j(j+1)\hbar^2$ is the eigenvalue of $\vec{\mathbf{J}}^2$) on your homework, for problem 12.5.2 on page 329 of your text.

   (a) (10 pts) For j=1, evaluate the three matrices $\mathbf{S}_i$ defined via:

   $$\mathbf{J}_i \doteq \hbar\mathbf{S}_i,$$

   and also evaluate the three matrices $\mathbf{S}_i^2$ and the three matrices $\mathbf{S}_i^3$. Is there a simplifying relationship between $\mathbf{S}_i^3$ and $\mathbf{S}_i$?

   (b) (15 pts) Find the matrix that represents the unitary tranformation that describes the (passive) rotation by α shown in Figure 1. Assume $j = 1$.

Turn over...

4. (25 pts) Consider the operators $\mathbf{J}_\alpha$ and $\mathbf{J}_\beta$ defined by:

$$\begin{array}{rcrcl} \mathbf{J}_\alpha & = & \cos\gamma \mathbf{J}_x & + & \sin\gamma \mathbf{J}_y \\ \mathbf{J}_\beta & = & -\sin\gamma \mathbf{J}_x & + & \cos\gamma \mathbf{J}_y \end{array} .$$

For all of the following, use the general state $|jm\rangle$, which is an eigenket of $\vec{\mathbf{J}}^2$ with eigenvalue $j(j+1)\hbar^2$ and of $\mathbf{J}_z$ with eigenvalue $m\hbar$.

(a) (5 pts) Evaluate $\langle \mathbf{J}_\alpha \rangle$ and $\langle \mathbf{J}_\beta \rangle$.

(b) (10 pts) Evaluate $\langle \mathbf{J}_\alpha^2 \rangle$ and $\langle \mathbf{J}_\beta^2 \rangle$.

(c) (10 pts) Evaluate whether $\Delta J_\alpha \Delta J_\beta$ satisfy the uncertainty principal in this form:

$$(\Delta J_\alpha)^2 (\Delta J_\beta)^2 \geq \frac{1}{4}\langle [\hat{\mathbf{J}}_\alpha, \hat{\mathbf{J}}_\beta]_+ \rangle^2 + \frac{1}{4} |\langle [\mathbf{J}_\alpha, \mathbf{J}_\beta] \rangle|^2$$

5. (20 pts) Consider a particle in a state described by

$$\psi = N(x + y + 2z)e^{-\beta r}$$

where N is a normalization factor and β is a real-valued parameter. The component of angular momentum l_z' about the z' axis, where the z' axis is rotated toward the x axis by an angle α as shown in Fig. 1, is measured. You can assume that components in the two systems satisfy:

$$\begin{array}{rcl} x & = & x' \cos\alpha + z' \sin\alpha \\ y & = & y' \\ z & = & z' \cos\alpha - x' \sin\alpha \end{array}$$

(a) (10 pts) Find each of the probabilities $P(l_z' = 0)$, $P(l_z' = +\hbar)$, and $P(l_z' = -\hbar)$.

(b) (10 pts) Determine the value(s) of $\tan\alpha$ that minimize each of $P(l_z' = 0)$, $P(l_z' = +\hbar)$, and $P(l_z' = -\hbar)$.

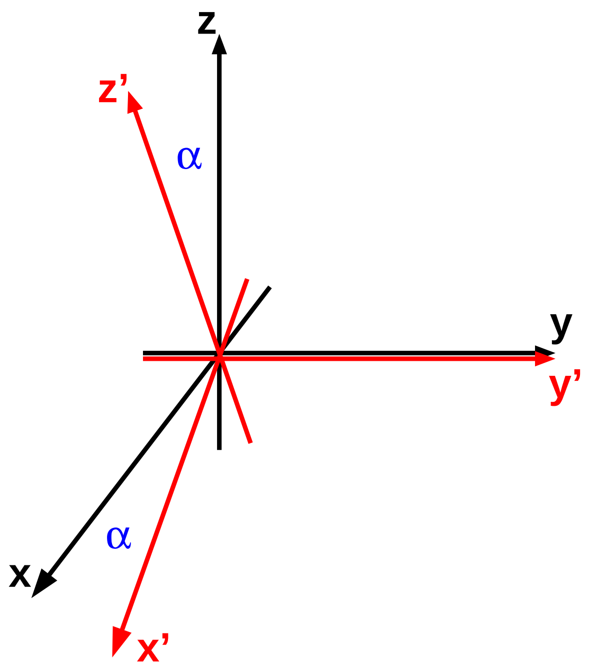

Figure 1: For use in problems 3 and 5.