4(cont) If $\langle x \rangle = \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{2m\omega}}$

Then have $\frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{2m\omega}} = \sqrt{\frac{\hbar}{2m\omega}}\cos\theta_1$

$\Rightarrow \cos\theta_1 = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\theta_1 = \pi/4}$

d) $|\psi(0)\rangle = \frac{1}{\sqrt{2}}|u_0\rangle + \frac{1}{\sqrt{2}}e^{i\pi/4}|u_1\rangle$

$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle = \frac{1}{\sqrt{2}}e^{-i\omega t/2}|u_0\rangle + \frac{1}{\sqrt{2}}e^{i\pi/4}e^{-i3\omega t/2}|u_1\rangle$

$|\psi(t)\rangle = e^{-i\omega t/2}\left(\frac{1}{\sqrt{2}}|u_0\rangle + \frac{1}{\sqrt{2}}e^{-i(\omega t - \pi/4)}|u_1\rangle\right)$

Overall phases don't matter. So $\boxed{\theta_1(t) = \frac{\pi}{4} - \omega t}$

Clearly the overall phase factor cancells out when computing $\langle x \rangle(t)$ so will get same answer as in part c) w/ $\theta_1 \to \theta_1(t)$ since this is only time dep.

$\therefore \boxed{\langle x \rangle(t) = \sqrt{\frac{\hbar}{2m\omega}}\cos(\pi/4 - \omega t)}$