4.13) ⇒ Prob. of measuring energy E_m is $|\langle u_m|\psi(t)\rangle|^2 = |C_m|^2$

Prob. measurement of energy yields $E > 2\hbar\omega$ = Prob. of measuring E_m w/ $m \geq 2$

$$= \sum_{m=2}^{\infty} |C_m|^2$$

So $\boxed{\mathcal{P} = \sum_{m=2}^{\infty} |C_m|^2}$

When $\mathcal{P} = 0$, $C_m, m \geq 2 = 0$ ⇒ Only C_0 and C_1 are not necessarily 0

b) Only $C_0, C_1 \neq 0$ ⇒ $|\psi(0)\rangle = C_0|u_0\rangle + C_1|u_1\rangle$

Normalization:

$$1 = \langle\psi(0)|\psi(0)\rangle = (\langle u_0|C_0^* + \langle u_1|C_1^*)(C_0|u_0\rangle + C_1|u_1\rangle)$$

$$\boxed{1 = |C_0|^2 + |C_1|^2}$$

$$\langle H\rangle(t=0) = \langle\psi(0)|H|\psi(0)\rangle = \langle\psi(0)|H(C_0|u_0\rangle + C_1|u_1\rangle) = \langle\psi(0)|(C_0\tfrac{1}{2}\hbar\omega|u_0\rangle + C_1\tfrac{3}{2}\hbar\omega|u_1\rangle)$$

$$= |C_0|^2\tfrac{1}{2}\hbar\omega + |C_1|^2\tfrac{3}{2}\hbar\omega = \boxed{\tfrac{1}{2}\hbar\omega(|C_0|^2 + 3|C_1|^2) = \langle H\rangle(t=0)}$$

IF $\langle H\rangle = \hbar\omega$ then

$$\hbar\omega = \tfrac{1}{2}\hbar\omega(|C_0|^2 + 3|C_1|^2) \Rightarrow 2 = |C_0|^2 + 3|C_1|^2$$

$$\Rightarrow 2 = (1 - |C_1|^2) + 3|C_1|^2 \Rightarrow 2 = 1 + 2|C_1|^2 \Rightarrow |C_1|^2 = 1/2$$

and $|C_0|^2 = 1 - |C_1|^2 = 1 - 1/2 = 1/2$

So: $\boxed{|C_0|^2 = |C_1|^2 = 1/2}$

c) Given $\langle H\rangle = \hbar\omega$ ⇒ $|C_0|^2 = |C_1|^2 = 1/2$

Choose C_0 real, positive ⇒ $C_0 = 1/\sqrt{2}$

Write C_1 as: $C_1 = 1/\sqrt{2}\, e^{i\theta_1}$, defining θ_1

So: $\langle x\rangle = \langle\psi(0)|x|\psi(0)\rangle = (\frac{1}{\sqrt{2}}\langle u_0| + \frac{1}{\sqrt{2}}e^{-i\theta_1}\langle u_1|) \times (\frac{1}{\sqrt{2}}|u_0\rangle + \frac{1}{\sqrt{2}}e^{i\theta_1}|u_1\rangle)$

$$= \tfrac{1}{2}\{\underbrace{\langle u_0|x|u_0\rangle}_{=0} + \underbrace{\langle u_1|x|u_1\rangle}_{=0} + e^{-i\theta_1}\underbrace{\langle u_1|x|u_0\rangle}_{=\sqrt{\frac{\hbar}{2m\omega}}} + e^{i\theta_1}\underbrace{\langle u_0|x|u_1\rangle}_{=\sqrt{\frac{\hbar}{2m\omega}}}\}$$ by prob. 2

$$= \sqrt{\frac{\hbar}{2m\omega}}\,\frac{1}{2}(e^{i\theta_1} + e^{-i\theta_1}) = \sqrt{\frac{\hbar}{2m\omega}}\cos\theta_1$$