1. I can write any operator in some representation. In this representation, the operator becomes analogous (in sense explained in (b)) to a matrix of complex values. The operation of taking the Hermitian conjugate (or †) of an operator becomes, in the representation, the operation of transpose-conjugation (i.e. T^*). Since operator / representation are analogous pictures, they have same properties $\Rightarrow$ I can just claim $\dagger = T^*$ always. How this works is illustrated in appendix B where it shows $(AB)^\dagger = B^\dagger A^\dagger$ by entering a representation, doing T^*, and then going back to the abstract operator notation.

a) $(aA+bB)^\dagger = (aA+bB)^{T*} = ((aA)^{T*} + (bB)^{T*})$

$= ((A^T a^T)^* + (B^T b^T)^*) = ((aA^T)^* + (bB^T)^*)$

$= (a^* A^{T*} + b^* B^{T*}) = (a^* A^\dagger + b^* B^\dagger)$ ✓

b) $(AB)^\dagger = (AB)^{T*} = (B^T A^T)^* = B^{T*} A^{T*} = B^\dagger A^\dagger$ ✓

2. Use result of 1a,b) above:

a) $[i(AB-BA)]^\dagger = i^*(AB)^\dagger + (-i)^*(BA)^\dagger$ by 1a)

$= -i(B^\dagger A^\dagger) + (i)(A^\dagger B^\dagger)$ by 1b)

$= i(A^\dagger B^\dagger - B^\dagger A^\dagger)$ algebra

$= i(AB - BA)$ $A = A^\dagger$, $B = B^\dagger$

since $O_p = O_p^\dagger$, it is Hermitian

b) This is just a) w/out the i in front $\Rightarrow$ will get $O_p^\dagger = -O_p \Rightarrow$ it is not Hermitian

c) $\left[\frac{1}{2}(AB+BA)\right]^\dagger = (\frac{1}{2})^*(AB)^\dagger + (\frac{1}{2})^*(BA)^\dagger$

$= \frac{1}{2}(B^\dagger A^\dagger) + \frac{1}{2}(A^\dagger B^\dagger)$

$= \frac{1}{2}(A^\dagger B^\dagger + B^\dagger A^\dagger) = \frac{1}{2}(AB+BA)$

since $O_p = O_p^\dagger$, it is Hermitian