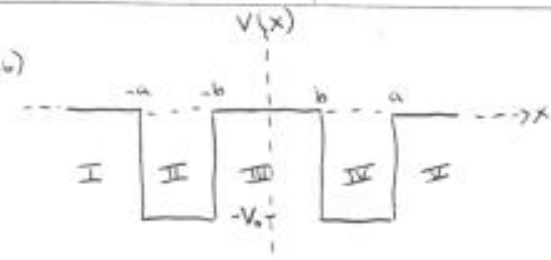
2. (S.16)



Bound state $\Rightarrow \quad -V_0 < E < 0$

so $\quad -E > 0$

$V_0 + E > 0$

Schrod. Eq.: $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V(x)-E)\psi$

In Regions II, IV have $V(x) = -V_0$

$$\Rightarrow \frac{d^2}{dx^2}\psi_{II,IV} = -q^2\psi_{II,IV} \quad \text{w/ } q^2 \equiv \frac{2m}{\hbar^2}(V_0+E) \qquad (1)$$

In Regions I, III, V have $V(x) = 0$

$$\Rightarrow \frac{d^2}{dx^2}\psi_{I,III,V} = \alpha^2\psi_{I,III,V} \quad \text{w/ } \alpha^2 \equiv \frac{-2m}{\hbar^2}E \qquad (2)$$

Note: $V(x) = V(-x) \Rightarrow [H_{op}, \mathbb{P}_{op}] = 0 \Rightarrow$ Eigenstates of H_{op} are eigenstates of parity as well. Treat the even/odd states separately:

Even parity eigenstates:

$\Rightarrow \psi(x) = \psi(-x)$

or: $\psi_I(-x) = \psi_V(x)$; $\psi_{II}(-x) = \psi_{IV}(x)$; $\psi_{III}(-x) = \psi_{III}(x)$

Find ψ_I: Soln of (2) is $\psi_I(x) = Ie^{-\alpha x} + I'e^{\alpha x}$

$\psi_I(x \to \infty) = 0 \Rightarrow I' = 0$

$\Rightarrow \psi_I(x) = Ie^{-\alpha x}$

Find ψ_{III}: Soln of (1) is $\psi_{IV}(x) = G\sin qx + H\cos qx$

Find ψ_{III}: Soln of (2) is $\psi_{III}(x) = Ee^{\alpha x} + Fe^{-\alpha x}$

even $\psi(x) \Rightarrow \psi_{III}(x)$ even $\Rightarrow E = F$

$\Rightarrow \psi_{III}(x) = F(e^{\alpha x} + e^{-\alpha x})$

Find ψ_{II}: $\psi_{II}(-x) = \psi_{IV}(x) \Rightarrow \psi_{II}(x) = -G\sin qx + H\cos qx$

Find ψ_I: $\psi_I(-x) = \psi_V(x) \Rightarrow \psi_I(x) = Ie^{\alpha x}$

Match soln's at $x = a$:

$$\psi_{IV}(a) = \psi_{II}(a) \Rightarrow G\sin qa + H\cos qa = Ie^{-\alpha a} \qquad (3)$$