UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 115A
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Midterm (continued)

3. Continued...

(e) Find an equation which could be solved to find the possible values of E in terms of V_0 and a.

$x=a \quad \psi_{II}(a) = \psi_{III}(a) \quad (1) \quad A e^{-\rho a} = B \sin(ka)$

$x=a, \left.\frac{\partial \psi_{II}}{\partial x}\right|_{x=a} = \left.\frac{\partial \psi_{III}}{\partial x}\right|_{x=a} \quad (2) \quad -\rho A e^{-\rho a} = kB\cos(ka)$

$$\frac{(2)}{(1)} \longrightarrow -\rho = k\frac{\cos(ka)}{\sin(ka)}$$

substituting for ρ & k

$$\boxed{-\sqrt{\frac{2m}{\hbar^2}(V_0 - E)} = \sqrt{\frac{2m}{\hbar^2}E}\,\mathrm{ctn}\left(\sqrt{\frac{2mE}{\hbar^2}}\,a\right)}$$

(f) For what combination of values of a and V_0 is there just barely one bound state ($E \sim V_0$)?

if $E \sim V_0$ the left hand side of the above eqn $= 0$. Then $\mathrm{ctn}(\,) = 0$

or $ka = \frac{\pi}{2} \longrightarrow \sqrt{\frac{2mE}{\hbar^2}}\,a = \frac{\pi}{2}$

Since $E \sim V_0$, $\left(\frac{2mV_0}{\hbar^2}\right)a^2 = \left(\frac{\pi}{2}\right)^2$

$$\boxed{V_0 a^2 = \frac{\left(\frac{\pi}{2}\right)^2 \hbar^2}{2m}}$$